Title: Are hyperbolic groups star-free?

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Abstract:

In the early 1980's, in the early days of the theory of automatic groups (which encodes certain finiteness properties of those groups using regular languages, that is, sets of strings recognised by finite state automata) questions started to be asked about what the structure of a regular language associated with a group might reveal about the group itself.

In particular semigroup theorists Rhodes and Margolis conjectured that the regular set of all geodesics of a (word) hyperbolic group must actually be a star-free set. The star-free sets form a natural, low complexity subclass of regular sets, and are defined to be those regular sets which can be expressed in terms of finite sets using only the operations of union, intersection, concatenation and complementation (but without the Kleene closure operation which is needed in addition to find expressions for the full range of regular sets).

Rhodes' and Margolis' conjecture is consistent with the low complexity of the solution of the word problem for these groups. (The word problem identifies the set of words which represent the identity element, and is an important logical problem arising from the work of Max Dehn.) Recently, Holt (Warwick), Hermiller (Nebraska) and I started to examine Margolis and Rhodes' conjecture.

We found it to be false, indeed we found a presentation of a free (and hence hyperbolic) group for which the set of geodesics is not star-free. But nontheless we can prove that certain small cancellation conditions on a presentation (which imply hyperbolicity) do make a group star-free, so Margolis and Rhodes were not far off.

I shall discuss these results, and more, starting with a general introduction, and assuming no background knowledge of finite state automata, regular languages, hyperbolic groups or small cancellation theory.