Yamashita no. 1

1. Borsuk-Ulam theorem

$\forall$ cont. distr. of temp & humid. on the earth, $\exists$ 2 locations opposite to each other & same temp & humid.

equ. fml: $\underset{\mathbb{Z}/2}{\overset{\circlearrowleft \text{"opos"}}{S^2}} \to \underset{\mathbb{Z}/2}{\overset{\circlearrowleft -1 \times}{\mathbb{R}^2}}$, $x \mapsto \begin{pmatrix} f_{temp}(x) - f_{temp}(\text{opos. } x) \\ f_{humid}(x) - f_{humid}(\text{opos. } x) \end{pmatrix}$

has zero somewhere.

eqv. fml: no $\mathbb{Z}/2$-equivar cont map $S^2 \to S^1$.

Generally: $\mathbb{Z}/2 \overset{-1 \times}{\curvearrowright} S^{n-1} \subset \mathbb{R}^n$

B-U thm: If $m > n$, no $\mathbb{Z}/2$ equvr cont hom $S^m \to S^n$.

(eqv. fml. $\mathbb{Z}/2$-equvr $S^n \to \mathbb{R}^n$ has zero)

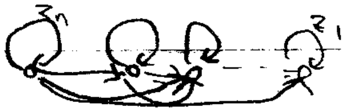
2. Quantum spheres.

- Odd-dim: $0 \le q \le 1$ $C(S_q^{2n-1})$ gen'd by $z_1 \cdots z_n$

$z_j z_i = q z_i z_j \ (i<j)$, $z_j^* z_i = q z_i z_j^* \ (i \ne j)$

$z_i^* z_i = z_i z_i^* + (1-q) \sum_{j>i} z_j z_j^*$, $\sum z_i z_i^* = 1$.

facts
- $q = 1$: $S^{2n-1} \subset \mathbb{C}^n$
- $q < 1$: (Hong-Szymanski) $C(S_q^{2n-1}) \simeq C(S_0^{2n-1})$

graph alg pres  n verts.

- Even-dim $C(S_q^{2n}) = C(S_q^{2n+1}) / \langle z_{n+1} - z_{n+1}^* \rangle$

similar facts.

Antipodal action $\mathbb{Z}/2 \curvearrowright C(S_q^m)$, $z_i \mapsto -z_i$

Thm (Y.) If $m > n$, no $\mathbb{Z}/2$-equivar unital $*$-hom $C(S_0^n) \to C(S_q^m)$ (posed by Baum-Hajac) schema $S^m_0 \xrightarrow{S^m_q} S^m$

comb graph alg — geom - index theory.

3. Equivariant quantization.

(valid for $G_q/K^{S,L}_q$: cpt simple G, Poisson-Lie $K^{S,L} < G$)

$S^{2n-1}_q = SU_q(n)/SU_q(n-1)$

$\curvearrowleft$ max torus $U(1)^{n-1}$ left. transl $\curvearrowright S^{2n-1}_q$

$U(1) = U(1)^{n-1}/U(1)^{n-2}$ right $\curvearrowright S^{2n-1}_q$.

(up to chg of par.) $(\lambda_1, \dots, \lambda_n).z_i = \lambda_i z_i$

Thm (Y.) $0 \leq q < 1$ $C(S^{2n-1}_q) \underset{KK^{U(1)^n}}{\simeq} C(S^{2n-1})$

Pf. 1. Cont field str. $(C(S^{2n-1}_q))_{0 \leq q \leq 1}$.

2. Compos. series $0 = J_n \subset J_{n-1} \subset \dots \subset J_0 \subset J_{-1} = C(S^{2n-1}_q)$

Soibelman-Vaksman, Nagy, Neshveyev-Tuset.

s.t. $J_{k-1}/J_k = C(U(1)) \otimes \mathcal{K}^{\otimes k}$.

$U(1)^n$ acts by : transl. on $U(1)$, gauge action on $\mathcal{K}(\ell_2 \mathbb{N})$.

compos w/ surjs $U(1)^n \to U(1)$ from root data.

3. $ev_q : \Gamma([0,1], J_m^{(\bullet)}) \to J_m^{(q)}$ is $KK^{U(1)^n}$-equiv.

(red. to sub q, Toeplitz quant).

4. Index

Thm (Y.) finite group $G \curvearrowright M$ cpt mfd free action

$\forall\, x \in KK^G(C(M), C(M))$, $\mathrm{Tr}\, x|_{K^0 M} - \mathrm{Tr}\, x|_{K^1 M} \in |G|\mathbb{Z}$

Pf. (Emerson-Meyer) $\mathrm{Tr}\, x|_{K^0} - \mathrm{Tr}\, x|_{K^1}$ is

(Kasparov) $\Delta \otimes (x \otimes 1_{C\ell M}) \otimes m \otimes D \in KK(\mathbb{C}, \mathbb{C})$

$\Delta \in KK(\mathbb{C}, C(M) \otimes \mathbb{C}\ell(M))$, $m : C(M) \otimes \mathbb{C}\ell(M) \to \mathbb{C}\ell(M)$.

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$D \in KK(\mathbb{C}\ell(M), \mathbb{C})$. "Euler char op"

$\xi_x = \Delta \otimes (x \otimes 2) \otimes M \in K(\mathbb{C}\ell(M))$

G-inv. metric $\rightsquigarrow \xi_x \in K_0^G(\mathbb{C}\ell(M))$. $D \in K_G^0(\mathbb{C}\ell(M))$

$\xi_x \otimes D = \mathrm{Ind}\, D_{\xi_x} = |G|\, \mathrm{Ind}\, D_{\xi_x}/G \in |G|\,\mathbb{Z}$.

Pf. of q-B U.

$\pi : C(S_q^{2n+1}) \to C(S_q^{2n})$ not surj.

suppose $\phi : C(S_q^{2n}) \to C(S_q^{2n+1})$ $\mathbb{Z}/2$-equiv

$\rightsquigarrow [\phi \circ \pi] \in KK^{\mathbb{Z}/2}(C(S_q^{2n+1}), C(S_q^{2n+1}))$

$$\begin{array}{c} \cong \\ KK^{\mathbb{Z}/2}(C(S^{2n+1}), C(S^{2n+1})) \end{array}$$

$\rightsquigarrow \mathrm{Tr}\, \phi \circ \pi|_{K_*} \in 2\mathbb{Z}$ but impossible from factoriz.