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# ÜBUNGEN ZUR VORLESUNG TOPOLOGIE II

Aufgabenblatt 7
Abgabe: Mittwoch, 10.6.2009 in der Vorlesung.

**Exercise 7.1.** Let $G = \langle x_1, \ldots, x_n | r_1, \ldots r_m \rangle$ be a finitely presented group. Show that $G^{ab}$ is the abelian group $\mathbb{Z}\{x_1, \ldots, x_n\}/(\rho_1, \ldots, \rho_n)$, where $\rho_i$ is obtained by "linearization" from the word $r_i$.

**Exercise 7.2.** Fill in the details of the proof that $F_g$ is a compact oriented differentiable manifold of dimension 2.

**Exercise 7.3.** Let $M$, $N$ be topological manifolds of dimension $n$. Let $M$ be compact and $N$ be connected. Let $f : M \to N$ be injective. Show that $f$ is surjective as well. Conclude that there is no continuous injective map $f : \mathbb{R}^m \to \mathbb{R}^n$ when $m > n$.

**Exercise 7.4.** Let $F_g$ be a closed oriented surface of genus $g \geq 0$. Let $D \subset F_g$ be a closed disc. Let $M$ be the Möbius band, as described in exercise 6.1, with boundary $\partial M$. Choose a homeomorphism $h : \partial D \cong \partial M$. Let $X_g := (F_g \setminus D) \coprod M/ \sim$, where $\sim$ is the equivalence relation induced by $h$. $X_g$ is a compact 2-manifold that is not orientable (do not show that here). The task is to compute $\pi_1(X)$ and $H_*(X)$.

Instead of one disc $D$, take two disjoint discs $D_1, D_2 \subset F_g$ and glue in two Möbius bands into the holes of $F_g \setminus (D_1 \cup D_2)$ and call the result $Y_g$. Compute $\pi_1(Y_g)$ and $H_*(Y_g)$.

Background: one can show that any nonorientable compact 2-manifold is diffeomorphic to exactly one of $X_g$ or $Y_g$.

**Exercise 7.5.** Let $A$ be a real nonsingular $n \times n$-matrix. Assume that $a_{ij} \geq 0$ for all $i, j$. Show that $A$ has a real eigenvector $v = (v_1, \ldots v_n)$ with $v_i \geq 0$ for all $i$ (Hint: Brouwers fixed point theorem).