ÜBUNGEN ZUR VORLESUNG TOPOLOGIE II

Aufgabenblatt 8
Abgabe: Mittwoch, 17.6., in der Vorlesung

Exercise 8.1. Let X be a finite CW-complex and let $f: Y \to X$ be a covering. Introduce a CW-structure on Y such that f becomes a cellular map.

Exercise 8.2. Let X, Y be finite CW-complexes. Show that the product $X \times Y$ is a CW-complex.

Exercise 8.3. Let X be a CW-complex and $Y \subset X$ be a subcomplex. Show that X/Y is a CW-complex.

Exercise 8.4. The projective spaces can be written in another way. Let $\mathbb{K}$ be one of the fields $\mathbb{R}, \mathbb{C}$. Then $\mathbb{KP}^n$ is the quotient of the space $\mathbb{K}^{n+1} \setminus \{0\}$ by the equivalence relation $v \sim av$ for all $v \in \mathbb{K}^{n+1} \setminus \{0\}$ and $a \in \mathbb{K} \setminus \{0\}$. Hence the inclusion $\mathbb{R} \to \mathbb{C}$ induces a map $\mathbb{RP}^n \to \mathbb{CP}^n$. Show that this map is cellular with respect to the CW-structures given in the lecture. Show that the conjugation on $\mathbb{C}$ induces a cellular map from $\mathbb{CP}^n$ to itself.

Exercise 8.5. (Lens spaces) Let $m \in \mathbb{Z}$, $r_1, \ldots, r_n \in \{1, \ldots m-1\}$ be integers that are coprime to m. We write $\zeta := \exp(\frac{2\pi i}{m}) \in \mathbb{C}$. Let $\mathbb{Z}/m$ act on $\mathbb{S}^{2n-1} \subset \mathbb{C}^n$ via the formula $q \cdot (z_1, \ldots, z_n) := (\zeta^{qr_1} z_1, \ldots, \zeta^{qr_n} z_n)$. Show that this is a free action. The quotient $L_{m;r_1,\ldots,r_n} := \mathbb{S}^{2n-1}/(\mathbb{Z}/m)$ is called a *lens space.*

Construct a CW-structure on $L_{m;r_1,\ldots,r_n}$.