ÜBUNGEN ZUR VORLESUNG TOPOLOGIE II

Aufgabenblatt 9
Abgabe: Mittwoch, 24.6., in der Vorlesung

Exercise 9.1. Let A_0, A_1, A_2 be finitely generated abelian groups and let $0 \to A_2 \to A_1 \to A_0 \to 0$ be an exact sequence. Show that $\sum_{i=0}^{2}(-1)^i \operatorname{rank}(A_i) = 0$.

Let $0 \to C_n \xrightarrow{d} C_{n-1} \xrightarrow{d} \ldots \xrightarrow{d} C_1 \xrightarrow{d} C_0 \to 0$ be a chain complex, such that C_k is a finitely generated abelian group. Show that $\sum_{i=0}^{n}(-1)^i \operatorname{rank}(C_i) = \sum_{i=0}^{n}(-1)^i \operatorname{rank}(H_i(C_\bullet)$. Let X be a finite n-dimensional CW-complex. Show that the Euler number of X can be computed as $\chi(X) = \sum_{i=0}^{n} \sharp A_i$, where $\sharp A_i$ is the number of i-cells of X.

Exercise 9.2. Let F_1, F_2 be two free abelian groups and let $f: F_2 \to F_1$ be a group homomorphism. Construct a CW-complex X such that $C_\bullet^{cell}(X)$ is isomorphic to $\ldots 0 \to F_2 \xrightarrow{f} F_1 \xrightarrow{0} \mathbb{Z} \to 0$ (the group F_i sits in degree i) and compute the homology of X. Let A be an arbitrary abelian group. Show that there exists a 2-dimensional CW-complex $M(A;1)$ such that $H_1(M(A;1)) \cong A$ and $\tilde{H}_k(M(A;1)) = 0$ for $k \neq 1$ (Hint: use, without proof, the fact that a subgroup of a free abelian group is free).

Exercise 9.3. Let X be a finite connected CW-comple. For simplicity, assume that X has exactly one 0-cell. Show that the inclusion map $X^{(2)} \to X$ of the 2-skeleton induces an isomorphism of the fundamental groups. Hint: Seifert-van Kampen Theorem.

Exercise 9.4. Let X be a finite CW-complex. Give a presentation of $\pi_1(X)$ in terms of the sets of 1-and 2-cells and the attaching maps of the 2-cells (the resulting presentation has a generator for each 1-cell and a relation for each 2-cell of X). Use this presentation to show the Hurewicz Theorem. Hint: exercises 7.1 and 9.3.

Exercise 9.5. Compute the homology of the lens space $L_{m;r_1,\ldots,r_n}$ using the CW structure from exercise 8.5.