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Varsovian Models

(joint with Farmer & Grigor)

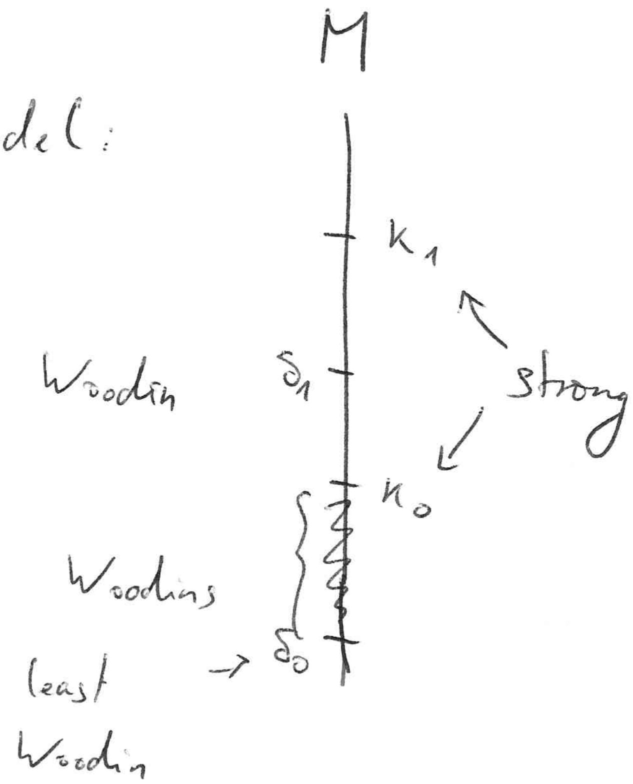
~~Let L[E] be a "nicely iterable~~

Is there an extender model $L[E]$ whose mantle $\mathbb{M}$ is not a $\underbrace{\text{fully iterable}}_{\substack{\text{in } L[E] \\ \text{or in itself}}}$ (hod) mouse?

Fix the following $L[E]$ model:

$$M = M_{swsw}$$

the least iterable class sized model with a Woodin between two strongs.



W is a ground for M iff $M = W[g]$ for some g $\mathbb{P}$-gen. $/ W$, $\mathbb{P} \in W$.

Mantle of M $\quad \mathbb{M} = \bigcap$ all grounds.

<u>Thm</u> (with Farmer and Grigor)

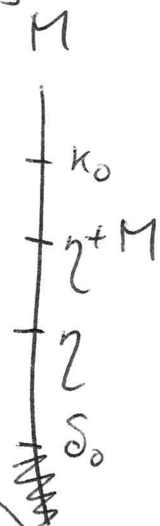
The mantle of M is a strategic extender model with 2 Woodin cardinals.

We assume that $M^\#$ exists.

1^{st} step:

Define V_0, the 1^{st} Varsovian model given by M.

Let T = the collection of all iteration trees $\mathcal{T}$ s.t. $\mathcal{T} \in M|\kappa_0$, $\mathcal{T}$ lives on $M|\delta_0$, ~~there is and from some point on~~ and all the extenders from $\mathcal{M}(\mathcal{T})$ satisfy the axioms associated with $\mathcal{M}(\mathcal{T})$'s extender algebra and $\delta(\mathcal{T})^* = \eta^+$, η a strong cutpoint.



attempting to make $M|\eta^+$ generic

Let $\Sigma \in V$ be the iteration strategy for M.

Let $\mathcal{T} \in T$.

$\mathcal{T}$ is according to Σ. Let $b = \Sigma(\mathcal{T})$.

$b \notin M$.

$$\mathcal{M}_b^{\mathcal{T}}, \mathcal{P} = \mathcal{P}[\mathcal{M} \restriction \delta(\mathcal{T})] = M.$$

We have $\mathcal{P} = \mathcal{M}_b^{\mathcal{T}}$.

The first directed system is:

$\mathcal{F}$, consisting of $\mathcal{P}$ = result of a $\mathcal{P}$-construction inside M above $\delta(\mathcal{T})$, $\mathcal{T} \in T$.

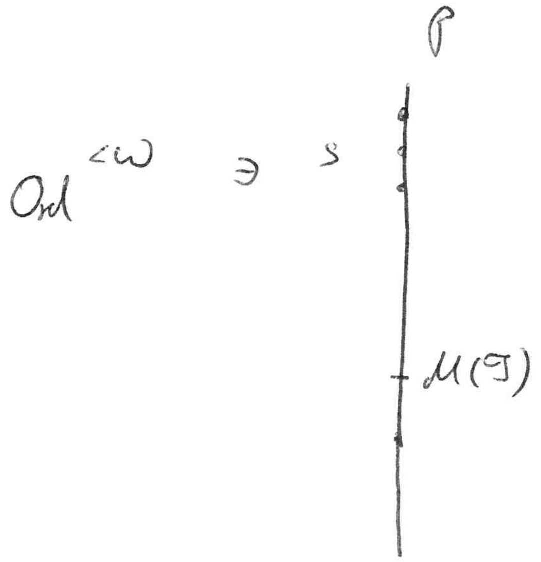
(as a matter of fact, all these $\mathcal{P}$'s are Σ-iterates of M).

with iteration maps $\pi_{\mathcal{P},\mathcal{P}'}$. ← in V

$\pi_{\mathcal{P},\mathcal{P}'}$ = iteration map from $\mathcal{P}$ to $\mathcal{P}'$ given by Σ.

$$(\mathcal{M}_\infty, \pi_{\mathcal{P},\infty} \mid \mathcal{P} \in \mathcal{F}) = \text{dir lim}\,(\mathcal{F}, \pi_{\mathcal{P},\mathcal{P}'} \mid \mathcal{P}, \mathcal{P}' \in \mathcal{F}, \mathcal{P} \trianglerighteq \mathcal{P}' \downarrow)$$

There is a system inside M "covering" this system.



P from the system is <u>s-iterable</u> iff for all $\mathcal{U} \in M|\kappa_0$, $\mathcal{U}$ lives on $\mathcal{M}(\mathcal{T}) = P|\delta_0^P$, $\mathcal{U}$ is maximal, $\exists$ cofinal branch $b \in M^{Coll(\omega, \max(s))}$ s.t.

a) $\pi^{\mathcal{U}}_{0,b}(s) = s$ and

b) $\pi^{\mathcal{U}}_{0,b}(P|\max(s)) = P(\mathcal{M}(\mathcal{U}))|\max(s)$.

P is <u>strongly s-iterable</u> iff P is s-iterable and for all $\mathcal{U}$ as above, any b as above moves

$$H^{P|\max(s)}(\gamma_s^P \cup \underset{\underset{s\setminus\{\max(s)\}}{\shortparallel}}{\bar{s}})$$ the same way.

$$= \min \{ \pi^{s}_{P,\infty}(\xi) \mid P \in \mathcal{F},\ s \in \mathrm{Ord}^{<\omega},\ P \text{ strongly sitenable},\ \xi \in \mathrm{dom}(\pi^{s}_{P,\infty}) \}$$

is definable in M.

<u>Definition.</u> V_0 = 1^{st} Varsovian model of M

$= L[\mathcal{M}_\infty, \xi \mapsto \xi^*]$

We'll prove:

- $V_0 = \mathrm{HOD}^{M^{\mathrm{Coll}(\omega,<\kappa_0)}}_{\mathcal{E}}$ $\qquad \mathcal{E}$ = all E' s.t. $E^M \sim E'$ ${}^{M}_{\Phi}$
- M knows $\Sigma_{\mathcal{M}_\infty}$, restricted to trees living on $\mathcal{M}_\infty | \delta_0^{\mathcal{M}_\infty}$.
- V_0 knows $\Sigma_{\mathcal{M}_\infty}$ — " — trees $\in V_0$ — " —
- M is generic over V_0.
- $V_0 = \bigcap <\kappa_0$ - grounds of M.

finally we'll reorganise V_0