Stefan.

# Varsovian models with more Woodin cardinals.

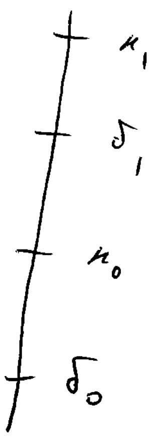
Let $\bar{M}$ be the sharp for a strong cardinal with unboundedly many Woodins below, and let $M$ be the result of iterating $\bar{M}$'s top measure out of the universe.

reminder: an inner model $W$ of $V$ is a ground of $V$ iff there is some forcing $\mathbb{P} \in W$ and some $g$ $\mathbb{P}$-generic$/W$ s.t. $W[g] = V$.

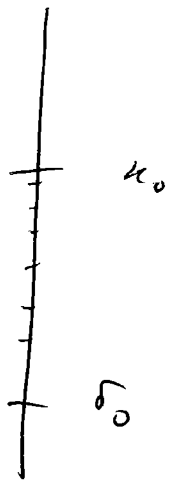
the mantle of $V$ is the intersection of all grounds, written $\mathbb{M}$.

goal: determine $\mathbb{M}^M$.

if $M = M_{swsw}$:



our current $M$:




Consider the following directed system:

$$(P, \pi_{P\bar{P}} : P, \bar{P} \in \mathcal{F}),$$

and $\mathcal{F}$ consists of models $P = P^M(m(\vec{\mathcal{U}}))$,

where $\vec{\mathcal{U}}$ is a seq. of normal trees

s.t. (roughly)

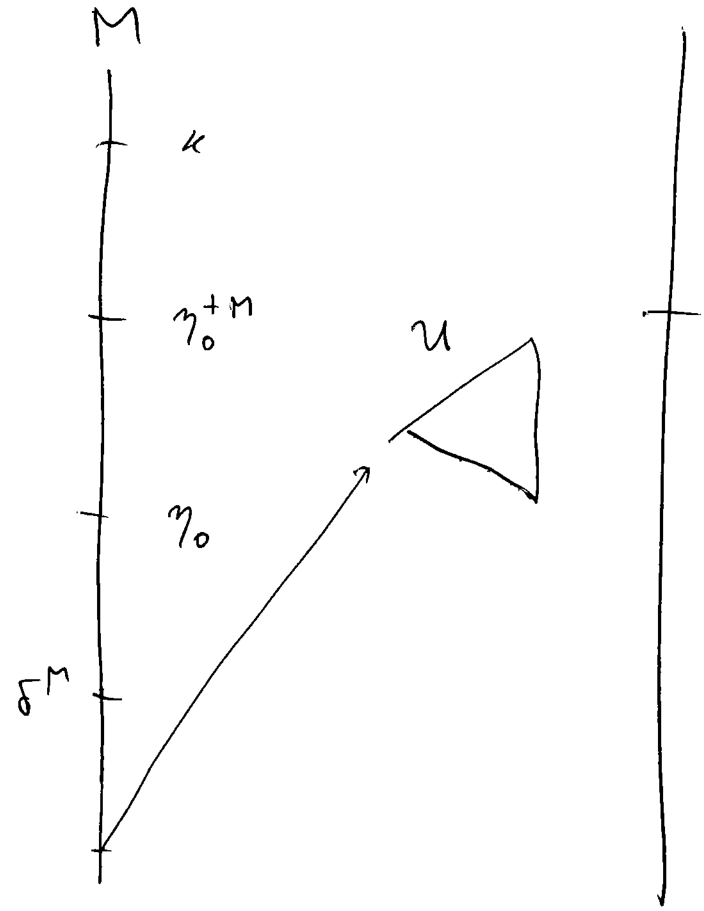
- $\mathcal{U}_0$ is on $M$ and lives below $\delta^M$ = the least woodin of $M$
- there is a seq. $\eta_0 < \dots < \eta_n < \kappa^M$ of cutpoints of $M$
- $lh(\mathcal{U}_k) = \eta_k^{+M} = \delta(\mathcal{U}_k)$

- $\mathcal{P}^M(\mathcal{M}(\mathcal{U}_k))$ is a proper class + has $\delta(\mathcal{U}_k)$ as a woodin cardinal
- if $k > 0$, $\mathcal{U}_{k-1}$ lives on $\mathcal{P}^M(\mathcal{M}(\mathcal{U}_{k-1}))$ + is below $\delta(\mathcal{U}_{k-1})$.

if $b$ = the true branch thru $\mathcal{U}$,

example:

$\mathcal{M}_b^{\mathcal{U}} = \mathcal{P}^M(\mathcal{M}(\mathcal{U}))$




$\delta(\mathcal{U}) = \eta_0^{+M}$

while the models $P \in \mathcal{F}$ are "visible" in $M$, the branches / embeddings are not. but we can approximate a system in $M$ by

$$\left( P, \hat{\pi}_{P, \bar{P}} : P, \bar{P} \in \mathcal{F} \right)$$

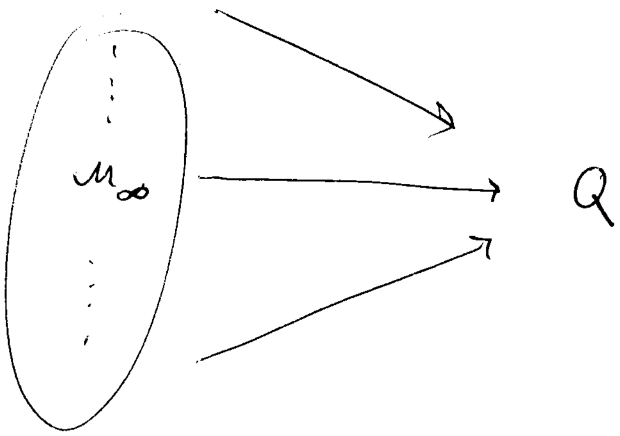
which yields the same direct lim, say $\mathcal{M}_\infty$.

<u>claim.</u> let $W$ be a ground of $M$. then $\mathcal{M}_\infty$ is definable in $W$.

why? fix a generic $g$ s.t. $W[g] = M$, fix $\lambda$ large enough + generic filters $h, \bar{h} \subset \mathrm{Col}(\omega, \lambda)$ s.t. $W[h] = M[\bar{h}]$.

$\mathcal{M}_\infty$ is fully iterable in $M$ and all of its generic extensions.

Coiterate in $W[h]$ all "similar" candidates for $\mathcal{M}_\infty$:



$Q \in W$. verify that for all suff. thin thick classes $\Gamma'$, $\Gamma$ in $W$,

$$\mathrm{Hull}^{Q}(\Gamma) \cong \mathcal{M}_\infty .$$

hence $\mathcal{M}_\infty$ is in $W$. $\dashv$

once again, let, for $\rho \in OR$,

$$\rho^* = \min\{\pi_{P,\infty}(\rho) : P \in \mathcal{I}\} .$$

the map $\rho \mapsto \rho^*$ is definable in $M$.

by the lem earlier in ralf's talk,

we now get

$$\mathcal{V} = L[\mathcal{M}_\infty, \rho \mapsto \rho^*]$$

is a ground of $M$.

can show that $\mathcal{V} \subset P$ for all $P \in \mathcal{F}$.

to see that $\mathcal{V}$ is the $\subset$-minimal grd of $M$, hence $\mathcal{V} = \mathbb{M}^M$, (hence also the bedrock), it now remains to be seen that $\rho \mapsto \rho^*$ is dy. th in all grnds.

and to see this, we need to see that for any given ground $W$ of $M$, $\mathcal{M}_\infty$ is fully it. th in $W$.

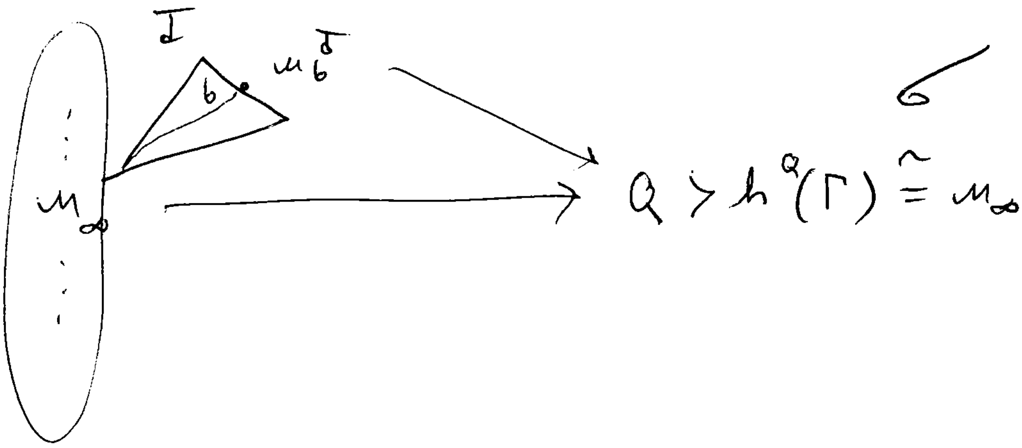
first let's see that $\mathcal{M}_\infty | \delta_\infty^{\mathcal{M}_\infty}$ is iterable in $W$.

fix $\lambda$ large enough + gen. filters

$h, \bar{h} \subset Col(\omega, \lambda)$ s.t.

$$W[h] = M[\bar{h}] .$$

in $W[h]$, contract all "suitable candidates for $M_\infty$ to a common iterate $Q$.




fix a tree $\mathcal{T}$ living on $M_\infty | \delta^{M_\infty}$ and arrange that for $b =$ the tree branch thru $\mathcal{T}$, there is an embedding

$$k : M_b^{\mathcal{T}} \longrightarrow Q$$

s.t. $k \circ i_b^{\mathcal{T}} \restriction M_\infty | \delta^{M_\infty} = \sigma \restriction M_\infty | \delta^{M_\infty}$

$\dashv$

since $p \mapsto p^*$ can be reconstructed from $(p \mapsto p^*) \upharpoonright \delta^{M_\infty}$, we actually get that $\mathcal{V}$ is the bedrock of $M$.

<u>remains:</u> $M_\infty$ is fully iterable in $M$ and all its gen. extensions.