martin zeman.

duality argument.

$$j : V \longrightarrow M \,, \qquad \kappa = \mathrm{crit}(j)$$

$$\mathbb{R} \in V \qquad j(\mathbb{R}) \in M \qquad \text{const, } \mathbb{R} \text{ usually s.t.}$$

$$\pi : j(\mathbb{R}) \longrightarrow \mathbb{R}$$

K generic for $\mathbb{R} / V$.

K' generic for $j(\mathbb{R})/_{\pi} K$.

then can lift

$$j : V[K] \longrightarrow \cancel{M} \; M[K']$$

$\mathcal{U}$ = the ultrafilter derived from j.

$\mathcal{U} \in V[K']$

$\dot{\mathcal{U}}$ a $j(\mathbb{R})/_{\pi} K$-name for $\mathcal{U}$.

<u>asme</u> the extension j is the ultrapower map by $\dot{\mathcal{U}}$.

define $\dot{I}$ on $V[K]$ on $\mathcal{P}(\kappa)$:

$$X \in \dot{I} \longleftrightarrow \Vdash^{V[K]}_{j(\mathbb{R})/_\pi K} X \notin \dot{U}$$

for $p \in j(\mathbb{R})/_\pi K$ we get

$$p \in K \iff j(f)(\kappa) \in j(K)$$

$$\iff \kappa \in j(\underbrace{\{\xi < \kappa : f(\xi) \in K\}})$$

$$\iff a_p \in \dot{U}^{K'} . \qquad a_p \in V[K]$$

$$e : j(\mathbb{R})/_\pi K \longrightarrow \mathcal{P}(\kappa) / \underline{I}$$

$$e(p) = [a_p]_I .$$

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ideal fest 2017 :

<u>question.</u> (eskew) can we have a precipitous ideal on ω_1 s.t. $j(\omega_1^V) = \omega_3^V$.

<u>a: yes.</u> can arrange this for NS_{ω_1} .

one can arrange:

if $j : V \longrightarrow M$, then if $S \subset \omega_3^V$ is stationary in M, then it is stationary in $V[G]$, G generic for $\mathcal{P}(\omega_1)/I$.

starting embedding: superstrong.

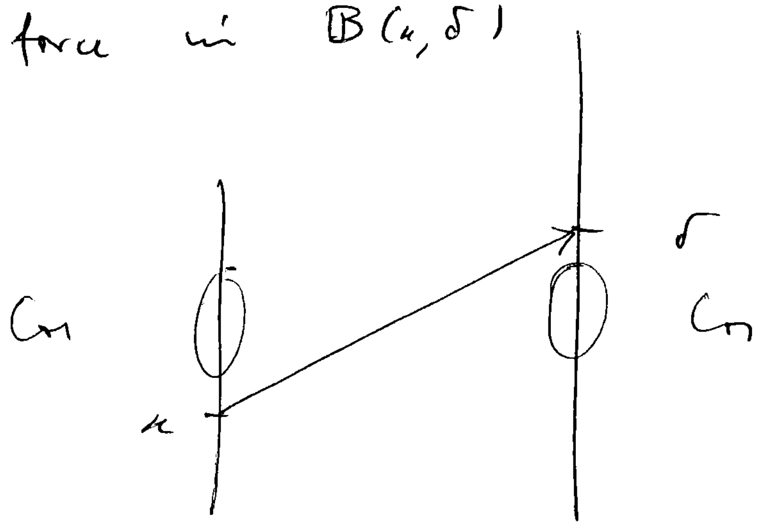
an ideal on κ is <u>strong</u> iff $j(\kappa) = \kappa^+$ when j is the generic embedding.

<u>proposition.</u> Let E be a superstrong (κ, δ) extender s.t. δ is strongly inaccessible. then in a generic extension we have $\kappa = \omega_1$, $\delta = \omega_3$ and an ideal I s.t. $j(\omega_1) = \omega_3$ when j is the generic embedding.

<u>proof</u>: Step 1. force with $\mathbb{P}$ which turns κ to ω_1.

step 2. force with $\mathrm{Col}(\kappa^+, < \delta)$. for $\delta = \omega_3$.

step 3. force wi $\mathbb{B}(\kappa, \delta)$



$V[K]$

need:

$j(\mathbb{P})$ nicely projects

to $\mathbb{P} * \mathrm{Col} * \mathbb{B}$.

step 1: first do $\mathrm{Col}(\omega, <\kappa)$. then at all $\alpha < \kappa$ s.t. $\mathbb{P}_\alpha \cap V_\alpha$ is a regular subposet of $\mathbb{P}_\alpha$.

force with $\mathrm{Col}(\alpha, <\kappa) \times \mathbb{B}(\alpha, \kappa)$ over $V^{\mathbb{P} \cap V_\alpha}$

in other words: if $p \in \mathbb{P}$, then $p(\alpha)$ is a $\mathbb{P}_\alpha \cap V_\alpha$ – name for $\mathrm{Col}(\alpha, <\kappa)$.

let G be a gen. for $\mathbb{P}$ over V.

H be a gen. for $\mathrm{Col}(\kappa^+, <\delta) / V[G]$.

$$j : V[G, H] \longrightarrow M[G', H'].$$

$\mathbb{B}(\kappa, \delta)$ = the $<\kappa$-supp. product of δ copies of $\mathrm{Add}(\kappa, 1)$. let L be a generic for $\mathbb{B}(\kappa, \delta) / V[G, H]$.

we have a nice projection $\pi : j(\mathbb{P}) \longrightarrow \mathbb{P}$ in M.
G' is a gen. for $j(\mathbb{P})/_{\pi} G$ over V.

$$j : V[G] \longrightarrow M[G'].$$

<u>step 2</u> (woodin)

H' = upward closure of $j'' H$.
for $j(\mathrm{Col}(\kappa^+, <\delta))$.
then $H' \in V[G']$ and is generic for $j(\mathrm{Col}(\kappa^+, <\delta)) / M[G']$.

~~now~~ $j : V[G, H] \longrightarrow M[G', H']$.

let L be a filter generic for $\mathbb{B}(\kappa, \delta)$ over $V[G, H]$. want to find L' generic for $j(\mathbb{B}(\kappa, \delta)) / M[G', H']$ s.t.

$j'' L \subset L'$.

for $p \in L$, let

$$m_p = j(p) \cup \{((j(\alpha), \kappa), \alpha) : (\alpha, \zeta) \in \text{dom}(p), \text{ some } \zeta\}$$

$p \in \mathbb{B}(\kappa, \delta)$, $\quad \text{dom}(p) \subset \delta \times \kappa$.

$$\mathcal{L} = \bigcup \{m_p : p \in L\}$$

$$\mathbb{B}' = j(\mathbb{B}(\kappa, \delta)) / \mathcal{L}.$$

<u>observation.</u> if L' is a poset for $\mathbb{B}'$ over $V[G', H']$, then L' is generic for $j(\mathbb{B}(\kappa, \delta))$ over $M[G', H']$.

<u>for this:</u> if $A \in M[G', H']$ is a maximal antichain in $j(\mathbb{B}(\kappa, \delta))$, then $A \cap \mathbb{B}'$ is a maximal antichain in $\mathbb{B}'$.

proof: based on the fact that $\mathbb{B}(\kappa, \delta)$ can be factored into pieces of size $< \kappa$. $\dashv$

this way for each $\alpha < \delta$ we have a fct. f in $V[G, H, L]$ s.t. $\alpha = j(f)(\kappa)$. hence every $x \in M[G', H', L']$ is of the form $j(g)(\kappa)$, for some $g : \kappa \longrightarrow M[G', H', L']$.