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In what follows, let M be a countable transitive model of ZFC, let $\mathbb{P} \in M$ be a partial order, and let g be $\mathbb{P}$-generic over M.

Problem 1. Let $\tau^g \in M[g]$, where $\tau \in M^{\mathbb{P}}$. Construct some $\sigma \in M^{\mathbb{P}}$ such that $\sigma^g = \bigcup \tau^g$.

Problem 2. Let $h \subset \mathbb{P}$ be a filter. Show that the following are equivalent.

(a) h is $\mathbb{P}$-generic over M.
(b) For all $A \in M$, if $A \subset \mathbb{P}$ is a maximal antichain, then $g \cap h \neq \emptyset$.
(c) For all $D \in M$, if $D \subset \mathbb{P}$ is predense, then $g \cap h \neq \emptyset$.

Problem 3. Let $c \subset \mathbb{C}$ be $\mathbb{C}$-generic over M. Let $f \colon \omega_1^M \to 2$, $f \in M[c]$ be such that $f \restriction \xi \in M$ for all $\xi < \omega_1^M$. Show that $f \in M$. (Hint. Let $f = \tau^g$. For each $\xi < \omega_1^M$, choose $p \in c$ and $f_\xi \in M$ with $p \Vdash \tau \restriction \check{\xi} = \check{f}_\xi$.)

Problem 4. Let κ be a regular cardinal in M such that M thinks that $\mathbb{P}$ has the κ-c.c. Let a be a set of ordinals in $M[g]$ of size less than κ in $M[g]$. Show that there is some $b \in M$ of size less than κ in M such that $b \supset a$.

Problem 5. (A generalization of Problem 3.) Let κ be a regular cardinal in M, κ being bigger than the size of the forcing $\mathbb{P}$ in M. Let A be a set of ordinals in $M[g]$ such that $A \cap a \in M$ for all $a \in M$ such that a has size less than κ in M. Show that $A \in M$.

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