Algebraic Geometry 2 | Prof. Dr. Urs Hartl
WiSe 2012/13 | Martin Brandenburg

Homework sheet 1

Due date: Monday, 15.10.12 at 12 noon

1. Let X, Y be topological spaces. We write $\hom(X, Y)$ for the set of morphims, i.e. continuous maps $X \to Y$.

   (a) Prove that $\underline{\hom}(X, Y)(U) := \hom(U, Y)$ defines a sheaf of sets $\underline{\hom}(X, Y)$ on X.

   (b) Prove the corresponding statement for morphisms between locally ringed spaces.

   Hint. This just means that one can glue morphisms between topological spaces, respectively between locally ringed spaces.

   (4 Punkte)

2. Let $\varphi : \mathcal{F} \to \mathcal{G}$ be a homomorphism between sheaves of sets on a topological space X. Prove the following statements:

   (a) The maps on stalks $\varphi_x : \mathcal{F}_x \to \mathcal{G}_x$ are injective for all $x \in X$ if and only if the maps $\varphi(U) : \mathcal{F}(U) \to \mathcal{G}(U)$ are injective for all open subsets $U \subseteq X$.

   (b) The corresponding statement with "bijective" instead of "injective" is also true.

   (c) The corresponding statement with "surjective" instead of "injective" is *false*. For this consider the sheaf $\mathcal{F} = \underline{\hom}(\mathbb{C}, \mathbb{C} \setminus \{0\})$ from exercise 1 and the homomorphism $\varphi : \mathcal{F} \to \mathcal{F},\ f \mapsto f^2$.

   (6 Punkte)

3. Let X be a topological space. Set $C_X := \underline{\hom}(X, \mathbb{R})$.

   (a) Prove that (X, C_X) is a locally ringed space.

   (b) To any continuous map $X \to Y$ construct a morphism of locally ringed spaces $(Y, C_Y) \to (X, C_X)$.

   (c) $\star$ Do all morphisms $(Y, C_Y) \to (X, C_X)$ arise in this way?

   (4+2* Punkte)

4. Let p be a prime. Describe the spectrum $\mathrm{Spec}(\mathbb{Z}_{(p)})$ as a topological space.

   (2 Punkte)