Algebraic Geometry 2 | Prof. Dr. Urs Hartl
WiSe 2012/13 | Martin Brandenburg

Homework sheet 5

Due date: Monday, 12.11.12 at 12 noon

1. Which elements of $\mathbb{Q}[\sqrt{5}]$ are integral over $\mathbb{Z}$? *(4 points)*

2. Let k be a field and $n \geq 1$.

   (a) Prove that $\mathbb{P}^n_k$ is proper over k.

   (b) Is $\mathbb{A}^n_k$ proper over k?

   Hint. You can use the valuative criterion. *(4 points)*

3. Let G be a group scheme over a field k and $n \in \mathbb{N}$.

   (a) Prove that for every k-scheme T the set $\hom_k(T, G)$ is endowed with a natural group structure. *Remark:* This is the only way how to produce an ordinary group out of a group scheme.

   (b) Determine these groups in the cases $G = \mathbb{G}_a, \mathbb{G}_m, \boldsymbol{\mu}_n$ explicitly.

   (c) $\star$ Find an affine group scheme G over k with the following property: For every k-algebra A we have $\hom_k(\mathrm{Spec}(A), G) \cong \mathrm{GL}_n(A)$.
   Remark: Because of this property one writes $G = \mathrm{GL}_n$.

   (4+2* points)

4. Let G be an ordinary group. Endow the scheme $(G)_k := \coprod_{g \in G} \mathrm{Spec}(k)$ with the structure of a group scheme over k. Then determine the group $\hom_k(T, (G)_k)$, when T is a connected k-scheme.

   (4 points)