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# Homework sheet 8

Due date: Monday, 3.12.12 at 12 noon

1. Let $p$ be a prime number and $k = \mathbb{F}_p$. Find the relative Frobenius morphism $X \to X^{(p)}$ for:

   (a) $X = \mathbb{A}_k^n$

   (b) $X = \mathbb{P}_k^n$

   (c) $X = \operatorname{Spec}(k(t))$

   *(4 points)*

Let $k$ be a field. For a homogeneous ideal $I \subseteq k[x_0, \ldots, x_n]$ let $V_{\mathbb{P}^n}(I) \subseteq \mathbb{P}_k^n$ be the corresponding projective $k$-scheme (from Example 1.2.13).
In the following exercises $k$ is assumed to be algebraically closed.

2. Let $d \in \mathbb{N}$ satisfy $d \in k^*$. Consider the fermat curve $X = V_{\mathbb{P}^2}(x^d + y^d - z^d)$. Let $\varphi : X \to \mathbb{P}^1$ be the morphism which is given on the function fields by $k(\mathbb{P}^1) = k(t) \to k(X)$, $t \mapsto \frac{y}{z}$. Find the degree, the ramification points as all as the ramification indices of $\varphi$.

   *Hint.* First compute $\varphi$ over the two affine charts of $\mathbb{P}^1$ and use the solution to exercise 4, sheet 6.

   *(4 points)*

3. Let $\varphi : \mathbb{P}^1 \to \mathbb{P}^1$ be the morphism corresponding to the homomorphism of function fields $\varphi^* : k(\mathbb{P}^1) = k(t) \to k(x) = k(\mathbb{P}^1)$, $t \mapsto x^2(x+1)^2$. Find the degree, the ramification points as all as the ramification indices of $\varphi$. Is $\varphi$ separable?

   *(4 points)*