Algebraic Geometry 2 — Prof. Dr. Urs Hartl
WiSe 2012/13 — Martin Brandenburg

Homework sheet 12

Due date: Monday, 14.1.13 at 12 noon

1. Let E be an elliptic curve over a perfect field k with $\mathrm{char}(k) \neq 2, 3$. Without loss of generality E is in Weierstraß normal form $y^2 - x^3 - a_4 x - a_6 = 0$. Compute the group $\mathrm{Aut}_k(E)$ in terms of $j(E)$ and k.

   (4 points)

2. For the following elliptic curves in Weierstraß normal form over k compute the groups $E(k)$ and $\mathrm{Aut}_k(E)$, as well as the numbers $\#\,\mathrm{Aut}_{k^{\mathrm{alg}}}(E)$ and $j(E)$.

   (a) $E : y^2 + y - x^3 = 0$ over $k = \mathbb{F}_2$. Also Compute $\#E(\mathbb{F}_4)$.

   (b) $E : y^2 - x^3 + x = 0$ over $k = \mathbb{F}_3$.

   (4 points)

3. Let E be an elliptic curve in Weierstraß normal form over a perfect field k. Find an explicit description of the doubling map $[2] : E \to E$ and compute the number of k^{alg}-valued points of its kernel $\ker[2] \subseteq E$. Differentiate the cases $\mathrm{char}(k) = 2$ and $\neq 2$.

   (4 points)