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Line Bundles on Rigid Analytic Spaces

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Abstract. Let K be a complete discretely valued field and X_K a smooth proper rigid analytic space over K, which has a strictly semi-stable formal model. We construct a smooth rigid analytic group variety that represents the Picard functor of line bundles on X_K on the category of smooth rigid analytic spaces. Its identity component is an extension of a smooth proper rigid analytic group by an affine torus. The Néron-Severi group of X_K is finitely generated.

1 The Main Result

This is a report on joint work with W. Lütkebohmert [HL]. Our intention in this article is to explain the main result on the representability of the Picard functor, Theorem 1.5. We restrict ourselves at some points to indicate reasons rather than give detailed proofs and we leave out technicalities which are dispensable. The reader can find complete proofs of all our statements in [HL].

Rigid analytic spaces were introduced by J. Tate at a seminar at Harvard in 1961 following ideas of A. Grothendieck. His notes were later published in the Inventiones [Ta]. In the 1970's M. Raynaud emphasized the relation between rigid analytic spaces and formal schemes [Ra]. Besides these two articles we refer the reader to [FRG] and [BGR] for an introduction to the subject.

Let K be a field of any characteristic which is assumed to be complete with respect to a non-archimedean discrete valuation of rank one and let R be its valuation ring. We denote by π a uniformizing parameter of R and by k the residue field of R. Moreover let X_K be a *smooth* rigid analytic space over K which is *proper* and *connected* and assume that there exists a K-rational point $x : \operatorname{Sp} K \to X_K$ of X_K. We want to study line bundles on X_K. A *line bundle* $\mathcal{L}_K$ on X_K is

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a locally free sheaf of rank one on X_K , i.e. there exists an admissible covering $\{U_K^i\}$ of X_K such that $\mathcal{L}_K|_{U_K^i} \cong \mathcal{O}_{U_K^i}$. Since A. Grothendieck the classification of line bundles is approached by means of the *relative Picard functor*

$$\underline{\mathrm{Pic}}_{X_K/K} \;:\; (\text{Rigid analytic spaces}) \longrightarrow (\text{Sets})\;, \quad V_K \longmapsto \underline{\mathrm{Pic}}_{X_K/K}(V_K)$$

where

$$\underline{\mathrm{Pic}}_{X_K/K}(V_K) \;:=\; \left\{ \mathrm{Isoclass}\,(\mathcal{L}_K,\lambda) : \begin{array}{l} \mathcal{L}_K \text{ line bundle on } X_K \times_K V_K \\ \lambda : \mathcal{O}_{V_K} \xrightarrow{\sim} (x \times \mathrm{id}_{V_K})^* \mathcal{L}_K \end{array} \right\}.$$

See the diagram below. To a morphism $f : V_K \longrightarrow W_K$ of rigid analytic spaces it assigns the morphism of sets

$$\underline{\mathrm{Pic}}_{X_K/K}(f) \colon \underline{\mathrm{Pic}}_{X_K/K}(W_K) \longrightarrow \underline{\mathrm{Pic}}_{X_K/K}(V_K)\,,\; (\mathcal{L}_K,\lambda) \mapsto (\mathrm{id}_{X_K} \times f)^*(\mathcal{L}_K,\lambda)\,.$$

So $\underline{\mathrm{Pic}}_{X_K/K}$ is a contravariant functor. It factors through the category of abelian groups. The isomorphism λ is called a *rigidification.* The purpose of the rigidification is twofold. Firstly it suppresses all line bundles coming from V_K and secondly it eliminates all automorphisms of $\mathcal{L}_K$. Namely an isomorphism of two rigidified line bundles $(\mathcal{L}_K,\lambda)$ and $(\mathcal{M}_K,\mu)$ on $X_K \times_K V_K$ is by definition an isomorphism $\varphi : \mathcal{L}_K \xrightarrow{\sim} \mathcal{M}_K$ satisfying

$$\mu \;=\; (x \times \mathrm{id}_{V_K})^* \varphi \circ \lambda : \;\; \mathcal{O}_{V_K} \xrightarrow{\sim} (x \times \mathrm{id}_{V_K})^* \mathcal{M}_K \,.$$

Thus $\mathrm{Aut}(\mathcal{L}_K,\lambda) = \{\mathrm{id}\}$. Equivalently we can describe the Picard functor as

$$\underline{\mathrm{Pic}}_{X_K/K}(V_K) \;=\; \mathrm{Pic}(X_K \times_K V_K) \;/\; (p \times \mathrm{id}_{V_K})^* \,\mathrm{Pic}(V_K)\;,$$

with $\mathrm{Pic}(Y_K)$ being the group of line bundles on the rigid analytic space Y_K , i.e. the *absolute Picard group* of Y_K , and $p : X_K \longrightarrow \mathrm{Sp}\, K$ the structural morphism. Namely using the section x of p we associate to every line bundle $\mathcal{L}_K \in \mathrm{Pic}(X_K \times_K V_K)$ the rigidified line bundle

$$\big(\mathcal{L}_K \otimes (p \times \mathrm{id}_{V_K})^*(x \times \mathrm{id}_{V_K})^* \mathcal{L}_K^{\vee} \;,\; \lambda\big)\;,$$

where $\lambda : \mathcal{O}_{V_K} \xrightarrow{\sim} (x \times \mathrm{id}_{V_K})^* \mathcal{L}_K \otimes (x \times \mathrm{id}_{V_K})^* \mathcal{L}_K^{\vee}$ is the canonical isomorphism. This shows that the Picard functor $\underline{\mathrm{Pic}}_{X_K/K}$ is independent of the K-rational point x .

We now ask whether the Picard functor is *representable.* This means that there exists a rigid analytic space Pic and an isomorphism of functors

$$\underline{\mathrm{Pic}}_{X_K/K} \;\cong\; \mathrm{Hom}_K(\,\bullet\,,\mathrm{Pic})\,.$$

Thus by the Yoneda-Lemma there is a line bundle $\mathcal{P}_K$ on $X_K \times_K \mathrm{Pic}$ together with a rigidification $\lambda_{\mathcal{P}} : \mathcal{O}_{\mathrm{Pic}} \xrightarrow{\sim} (x \times \mathrm{id}_{\mathrm{Pic}})^* \mathcal{P}_K$ such that for any rigid analytic space V_K and any rigidified line bundle $\mathcal{L}_K$ on $X_K \times_K V_K$ there is a unique morphism $f : V_K \longrightarrow \mathrm{Pic}$ and an isomorphism of rigidified line bundles $(\mathrm{id}_{X_K} \times f)^*(\mathcal{P}_K,\lambda_{\mathcal{P}}) \cong (\mathcal{L}_K,\lambda)$ on $X_K \times_K V_K$.

$$\begin{array}{ccccc} & & \mathcal{L}_K & & \mathcal{P}_K \\ & & / & \mathrm{id}_{X_K} \times f & / \\ X_K & \longleftarrow & X_K \times_K V_K & \longrightarrow & X_K \times_K \mathrm{Pic} \\ x \uparrow\downarrow & & x \times \mathrm{id}_{V_K} \uparrow\downarrow & & \downarrow\uparrow x \times \mathrm{id}_{\mathrm{Pic}} \\ & & & \exists ! f & \\ \mathrm{Sp}\, K & \longleftarrow & V_K & \dashrightarrow & \mathrm{Pic} \end{array}$$

In this article we want to show the representability of the Picard functor on the category of smooth rigid analytic spaces under the additional assumption that X_K *admits a strictly semi-stable formal model* X over R. We define this notion analogous to A.J. de Jong [dJ, Definition 2.16].

Definition 1.1 Let X be an admissible formal R-scheme (cf. [FRG, I]) and let X_0^σ for $\sigma = 1, \dots, s$ be the irreducible components of the special fiber X_0 of X. For each nonempty $M \subseteq N := \{1, \dots, s\}$ we define

$$X_0^M := \bigcap_{\sigma \in M} X_0^\sigma$$

as the scheme-theoretic intersection. X is called *strictly semi-stable over* R if

(a) X_0 is geometrically reduced,

(b) X_0^σ is a Cartier divisor on X for all $\sigma \in N$, i.e. X_0^σ is locally on X a principal divisor and

(c) X_0^M is smooth over k for all $M \subseteq N$ and equidimensional of dimension $\dim X - \# M$.

Remark 1.2 The conditions imply that the rigid analytic fiber X_{rig} is smooth over K; cf. Proposition 1.4.

Remark 1.3 Condition (b) implies that X is regular; cf. Proposition 1.4.

We can characterize strictly semi-stable formal R-schemes by the following proposition.

Proposition 1.4 *Let X be an admissible formal R-scheme. The following are equivalent:*

1.) *X is strictly semi-stable.*

2.) *Every closed point $x \in X_0$ of the special fiber admits an open neighborhood which for some $r \in \mathbb{N}$ is formally smooth over the formal scheme*

$$\mathrm{Spf}\, R \langle \xi_{i_1}, \dots, \xi_{i_s} \rangle / (\xi_{i_1} \cdot \dots \cdot \xi_{i_s} - \pi) .$$

Proof (cf. [dJ, 2.16]) The sufficiency of condition 2.) is clear. We show its necessity. So let X be strictly semi-stable and $x \in X_0$ a closed point of the special fiber. Say x lies on the irreducible components $X_0^{i_1}, \dots, X_0^{i_s}$ and not on the other components. On a neighborhood U of x the Cartier divisor X_0^σ is principal, say generated by ξ_σ. By (a) and (b) these generators satisfy the relation

$\xi_{i_1} \cdot \ldots \cdot \xi_{i_s} = u \cdot \pi$ for a unit u on U. After replacing ξ_{i_1} by $u \cdot \xi_{i_1}$, we may assume $u = 1$ and obtain a morphism

$$U \longrightarrow \operatorname{Spf} R\,\langle \xi_{i_1}, \ldots, \xi_{i_s} \rangle / (\xi_{i_1} \cdot \ldots \cdot \xi_{i_s} - \pi)\ .$$

By (c) the fiber $X^{\{i_1,\ldots,i_s\}}$ above the closed subscheme $\mathrm{V}(\xi_{i_1}, \ldots, \xi_{i_s})$ is smooth. This carries over to the formal completion along this closed subscheme. That in turn implies the formal smoothness of a whole neighborhood of x. □

There is still the conjecture that every quasi-compact smooth rigid analytic space admits a strictly semi-stable formal model after a suitable base ring extension. If the residue characteristic is zero, one can apply the stable reduction theorem of Mumford; cf. [TE, p. 198] which provides the good model. So the result is valid without further conditions in this case.

In general a formal geometry analog of A.J. de Jong's alterations result [dJ] is true. Namely, after a suitable finite separable field extension $\widetilde{K}$ of K, there exists an étale surjective morphism $\widetilde{X}_K \longrightarrow X_K$ such that $\widetilde{X}_K$ admits a strictly semi-stable model $\widetilde{X}$ over the valuation ring $\widetilde{R}$ of $\widetilde{K}$. Indeed one can write X_K locally as a smooth curve fibration. In [L2, Theorems 5.2 and 5.3] it was shown that étale locally on X_K this curve fibration can be embedded into a smooth projective curve fibration which has a semi-stable formal model. Then we apply the methods of de Jong to obtain the desired strictly semi-stable model of $\widetilde{X}_K$.

Now we are ready to state the main theorem.

Theorem 1.5 *Let X_K be a smooth rigid analytic space over K which is proper and connected and assume that there exists a K-rational point $x : \operatorname{Sp} K \to X_K$ of X_K. Further assume that X_K admits a strictly semi-stable formal model X over R.*

Then there exists a unique (up to canonical isomorphism) smooth rigid analytic group variety $\operatorname{Pic}_{X_K/K}$ and a morphism of functors

$$\Theta \;:\; \operatorname{Hom}_K(\;\bullet\;, \operatorname{Pic}_{X_K/K}) \longrightarrow \underline{\operatorname{Pic}}_{X_K/K}$$

which is universal in the following sense: For any smooth rigid analytic space V_K, the map

$$\Theta(V_K) \;:\; \operatorname{Hom}_K(V_K, \operatorname{Pic}_{X_K/K}) \xrightarrow{\;\sim\;} \underline{\operatorname{Pic}}_{X_K/K}(V_K)$$

is bijective. In particular, there exists a line bundle $\mathcal{P}_K$ on $X_K \times \operatorname{Pic}_{X_K/K}$ and an isomorphism $\lambda_{\mathcal{P}} : \mathcal{O}_{\operatorname{Pic}_{X_K/K}} \xrightarrow{\;\sim\;} (x \times \mathrm{id})^* \mathcal{P}_K$ such that, for any smooth rigid analytic space V_K and for any pair $(\mathcal{L}_K, \lambda) \in \underline{\operatorname{Pic}}_{X_K/K}(V_K)$, there is a unique morphism $f : V_K \longrightarrow \operatorname{Pic}_{X_K/K}$ and a unique isomorphism of rigidified line bundles $(\mathcal{L}_K, \lambda) \xrightarrow{\;\sim\;} (\mathrm{id} \times f)^*(\mathcal{P}_K, \lambda_{\mathcal{P}})$.

The identity component $\operatorname{Pic}^0_{X_K/K}$ of $\operatorname{Pic}_{X_K/K}$, i.e. the connected component containing the neutral element, is called the Picard variety *of X_K and $\mathcal{P}_K$ is called the* Poincaré bundle. *After a base field extension the Picard variety is an extension of an abeloid variety by an affine torus; an abeloid variety is a connected smooth rigid analytic group variety with proper underlying space.*

The Galois module $\mathrm{NS}_{X_K/K} := \operatorname{Pic}_{X_K/K}(\mathbb{K}) / \operatorname{Pic}^0_{X_K/K}(\mathbb{K})$ where $\mathbb{K}$ is the completion of an algebraic closure of K is called the Néron-Severi group *of X_K. The Néron-Severi group of X_K is finitely generated.*

Remark 1.6 (1) If X_K is the analytification of a proper algebraic variety, the rigid analytic Picard variety is the analytification of the classical algebraic Picard variety, due to the GAGA-principle; cf. [L1, Theorem 2.8].
(2) In the case of (1) the connected components of the Picard variety are proper as X_K is assumed to be smooth; cf. [FGA, n^{0}236, Theorem 2.1] or [BLR, Theorem 8.4/3].
(3) If X_K is a smooth connected proper rigid analytic group variety, the representability of $\underline{\mathrm{Pic}}_{X_K/K}$ was established by W. Lütkebohmert in [L2] resp. [BL]. In particular, he showed that $\mathrm{Pic}^0_{X_K/K}$ is smooth and proper and that the Néron-Severi group of X_K is finitely generated and torsion free.
(4) There are examples of (non-algebraic) proper smooth rigid analytic spaces where the Picard variety is not proper; cf. [Mu]. Let us briefly review the example. This is an analogue of the Hopf surface. It can be defined in the following way

$$X_K \;=\; (\mathbb{A}^2_K - \{0\})/\Gamma$$

where $\Gamma = \langle\gamma\rangle$ is generated by a single element γ which acts on $\mathbb{A}^2_K - \{0\}$ by

$$\gamma(\xi_1, \xi_2) \;=\; (\alpha_1\xi_1\,,\, \alpha_2\xi_2)$$

where $\alpha_1, \alpha_2 \in K^*$ with $0 < |\alpha_1| \le |\alpha_2| < 1$. One can show that X_K is smooth and proper and that it admits a strictly semi-stable formal model. One computes that $\mathrm{Pic}_{X_K/K} = \mathbb{G}_{m,K}$.

Proof. The rest of this article is devoted to the proof of this theorem. For technical reasons we first demonstrate the existence of the identity component of the representing space.

We introduce the category

$\mathfrak{C}_K$ of pointed rigid analytic spaces (V_K, v_K) where V_K is smooth and connected over K and where $v_K \in V_K(K)$ is a K-rational point. The morphisms in this category are the rigid analytic morphisms respecting the points.

Instead of the general Picard functor, we consider the slightly different functor

$$\underline{\mathrm{Pic}}^0_{X_K/K} \;:\; \mathfrak{C}_K \longrightarrow (\mathrm{Sets})\,, \quad (V_K, v_K) \longmapsto \underline{\mathrm{Pic}}^0_{X_K/K}(V_K, v_K)$$

of rigidified line bundles which are trivialized at the given point where

$$\underline{\mathrm{Pic}}^0_{X_K/K}(V_K, v_K) = \left\{ \mathrm{Isoclass}\ (\mathcal{L}_K, \lambda) : \begin{array}{c} \mathcal{L}_K \ \text{line bundle on}\ X_K \times_K V_K \\ \lambda : \mathcal{O}_{V_K} \xrightarrow{\ \sim\ } (x \times \mathrm{id}_{V_K})^*\mathcal{L}_K \\ (\mathrm{id}_{X_K} \times v_K)^*\mathcal{L}_K \cong \mathcal{O}_{X_K} \ \text{trivial} \end{array} \right\}.$$

These are in a sense the deformations of the trivial line bundle or phrased differently the line bundles which are "algebraically" equivalent to the trivial line bundle. Later on it turns out that the representing space of this functor is the identity component of the usual Picard functor.

We further show that the Néron-Severi group $\mathrm{NS}_{X_K/K}$ is finitely generated. Then we construct the rigid analytic Picard variety as the disjoint union

$$\mathrm{Pic}_{X_K/K} \;:=\; \coprod_{[\mathcal{L}_K] \in \mathrm{NS}_{X_K/K}} \mathcal{L}_K \otimes \mathrm{Pic}^0_{X_K/K}\ .$$

2 Typical Line Bundles

In order to show the representability of the functor $\underline{\mathrm{Pic}}^0_{X_K/K}$, i.e. to construct the Picard variety we want to introduce two typical types of line bundles.

1.) The first type of line bundle arises from the combinatorial configuration of the irreducible components of the special fiber X_0 . Let $n \in \mathrm{H}^1(X_0, \mathbb{Z})$ be a cohomology class. It can be represented by a cocycle $n = (n_{ij})$ with respect to an open covering $\{U^i\}$ of X , i.e. $n_{ij} \in \mathbb{Z}(U^i \cap U^j)$. Then for a coordinate function ξ on the multiplicative group $\mathbb{G}_{m,K}$ we obtain elements $\xi^{n_{ij}} \in \mathcal{O}^\times\big((U^i_{\mathrm{rig}} \cap U^j_{\mathrm{rig}}) \times_K \mathbb{G}_{m,K}\big)$ which clearly form a cocycle with respect to the covering $\{U^i_{\mathrm{rig}} \times_K \mathbb{G}_{m,K}\}$ of $X_K \times_K \mathbb{G}_{m,K}$. We consider the line bundle on $X_K \times_K \mathbb{G}_{m,K}$ which is given by this cocycle and denote it by (ξ^n) . We call line bundles of this type *multiplicative.*
2.) To any formal line bundle $\mathcal{L}$ on $X \times_R V$ we can consider the rigid analytic line bundle $\mathcal{L}_{\mathrm{rig}} := \mathcal{L} \otimes_R K$ on $X_K \times_K V_{\mathrm{rig}}$.

These two types of line bundles indicate that $\mathrm{Pic}^0_{X_K/K}$ should have a formal part and a torus part. In a way all line bundles on X_K are built from these two types. More precisely the following is true.

Proposition 2.1 *Let* V *be smooth over* $R\langle\zeta_1, \dots, \zeta_{t+1}\rangle/(\zeta_1 \cdot \ldots \cdot \zeta_{t+1} - \pi)$. *Let* $v \in V(R)$ *be a point above* $\{\zeta_1 = \ldots = \zeta_t = 1\}$. *Consider a rigid analytic line bundle* $\mathcal{L}_K$ *on* $X_K \times_K V_{\mathrm{rig}}$ *which is trivialized along* v .

Then $\mathcal{L}_K$ *is a tensor product* $\mathcal{L}_K \cong \mathcal{M}_K \otimes \mathcal{N}_{\mathrm{rig}}$ *of two line bundles where*

(1) $\mathcal{M}_K \cong (\zeta_1^{n_1}) \otimes \ldots \otimes (\zeta_t^{n_t})$ *is the multiplicative line bundle on* $X_K \times_K V_{\mathrm{rig}}$ *associated to suitable elements* $n_1, \dots, n_t \in \mathrm{H}^1(X_0, \mathbb{Z})$.
(2) $\mathcal{N}$ *is a formal line bundle on* $X \times_R V$ *which is trivialized along* v .

The decomposition $\mathcal{L}_K \cong \mathcal{M}_K \otimes \mathcal{N}_{\mathrm{rig}}$ *is unique. In particular, the Picard group of line bundles which are trivial along* v *decomposes into*

$$\mathrm{Pic}^0\big(X_K \times_K V_{\mathrm{rig}}\big) = \mathrm{H}^1(X_0, \mathbb{Z})^t \oplus \mathrm{Pic}^0(X \times_R V)\,.$$

Proof In order to prove the proposition we try to extend the line bundle $\mathcal{L}_K$ to a formal line bundle on $X \times_R V$. If $X \times_R V$ were regular, this would be possible. However that is not the case in general. On the other hand X possesses a covering by open sets U which are smooth over $\mathrm{Spf}\, R\langle\xi_1, \dots, \xi_s\rangle/(\xi_1 \cdot \ldots \cdot \xi_s - \pi)$. We can desingularize the product $U \times_R V$ by successive blowing-ups. Namely we blow up

$$\mathrm{Spf}\, R\langle\xi_1, \dots, \xi_s, \zeta_1, \dots, \zeta_{t+1}\rangle / \big(\xi_1 \cdot \ldots \cdot \xi_s - \pi\,,\ \zeta_1 \cdot \ldots \cdot \zeta_{t+1} - \pi\big)$$

in the open ideal (ξ_1, ζ_1) and obtain two charts of the blowing-up Y' :

- $\xi_1 = \zeta_1 \cdot \xi_1'$: The relations are equivalent to

$$\begin{aligned} \xi_1' \cdot \xi_2 \cdot \ldots \cdot \xi_s - \zeta_2 \cdot \ldots \cdot \zeta_{t+1} &= 0 \qquad \text{and} \\ \xi_1 \cdot \ldots \cdot \xi_s - \pi &= 0\,. \end{aligned}$$

Thus the number t was decreased by 1 while s stayed constant.

- $\zeta_1 = \xi_1 \cdot \zeta_1'$: The relations are equivalent to

$$\begin{aligned} \xi_2 \cdot \ldots \cdot \xi_s - \zeta_1' \cdot \zeta_2 \cdot \ldots \cdot \zeta_{t+1} &= 0 \qquad \text{and} \\ \xi_1 \cdot \ldots \cdot \xi_s - \pi &= 0\,. \end{aligned}$$

Thus the number s was decreased by 1 while t stayed constant.

By repeating this step we obtain a desingularization $U \times_R V \longleftarrow \ldots \longleftarrow Y^n$. Since Y^n is regular with $Y^n_{\mathrm{rig}} = (U \times_R V)_{\mathrm{rig}}$ the line bundle $\mathcal{L}_K$ extends to a formal line bundle $\mathcal{L}^{(n)}$ on Y^n . Further the above desingularization allows us to control the centers of the blowing-ups. A careful analysis shows that the line bundle $\mathcal{L}^{(n)}$ descends step by step to a line bundle $\mathcal{L}$ on $U \times_R V$ which extends the rigid analytic line bundle $\mathcal{L}_K$. We can further choose $\mathcal{L}$ to be trivialized along v .

Now we would like to glue the local extensions $\mathcal{L}$ and $\mathcal{L}'$ of our rigid analytic line bundle $\mathcal{L}_K$ above the intersection $(U \cap U') \times_R V =: W$. On the rigid analytic fiber W_{rig} we have an isomorphism

$$\psi_K : \mathcal{L}_{\mathrm{rig}} \xrightarrow{\sim} \mathcal{L}'_{\mathrm{rig}}.$$

In order to extend it to an isomorphism of the formal line bundles we have to study the invertible functions on W_{rig} . These obviously include all formal units $\mathcal{O}_W(W)^\times$ as well as $\xi_1, \ldots, \xi_s$ and $\zeta_1, \ldots, \zeta_{t+1}$. If W is connected it turns out that all invertible functions are built up from these

$$\mathcal{O}_{W_{\mathrm{rig}}}(W_{\mathrm{rig}})^\times = \mathcal{O}_W(W)^\times \oplus \Big((\xi_1^{\mathbb{Z}} \oplus \ldots \oplus \xi_s^{\mathbb{Z}}) + (\zeta_1^{\mathbb{Z}} \oplus \ldots \oplus \zeta_{t+1}^{\mathbb{Z}}) \Big).$$

The later sum is not direct. There is one relation $\xi_1 \cdot \ldots \cdot \xi_s = \pi = \zeta_1 \cdot \ldots \cdot \zeta_{t+1}$.

Since $\mathcal{L}$ and $\mathcal{L}'$ are both trivialized along v we see that there is no contribution of the ξ 's in ψ_K , i.e.

$$\psi_K = \zeta_1^{n_1} \cdot \ldots \cdot \zeta_t^{n_t} \cdot \psi$$

Since the collection of the ψ_K satisfies a cocycle condition the same is true for n_1 up to n_t and for ψ . So they form cocycles $n_1, \ldots, n_t \in \mathrm{H}^1(X_0, \mathbb{Z})$. The cocycle $(\zeta_1^{n_1} \cdot \ldots \cdot \zeta_t^{n_t})$ with respect to the covering consisting of the $U \times_R V$'s defines the multiplicative line bundle $\mathcal{M}_K := (\zeta_1^{n_1}) \otimes \ldots \otimes (\zeta_t^{n_t})$ on $X_K \times_K V_{\mathrm{rig}}$. In particular $\mathcal{M}_K|_{(U \times_R V)_{\mathrm{rig}}}$ is trivial. The collection of the isomorphisms $\psi : \mathcal{L} \xrightarrow{\sim} \mathcal{L}'$ forms a cocycle which allows us to glue the local extensions $\mathcal{L}$ to a formal line bundle $\mathcal{N}$ on $X \times_R V$. This yields the decomposition

$$\mathcal{L}_K \cong \mathcal{M}_K \otimes \mathcal{N}_{\mathrm{rig}}\,.$$

we are looking for. It is unique since the decomposition of the isomorphism was unique. □

3 The Construction of the Identity Component

We want to illustrate the construction of the Picard variety by discussing the example of the Tate curve. The Tate curve is the quotient $X_K = \mathbb{G}_{m,K}/q^{\mathbb{Z}}$ for some $q \in K^\times$ with $|q| < 1$. It is an algebraic curve, connected proper and smooth over K and admits a strictly semi-stable formal model X over R . The special fiber X_0 of this model is a closed chain of projective lines. Since X_K is an elliptic curve,

$\underline{\mathrm{Pic}}^0_{X_K/K}$ is represented by the Jacobi variety J_K of X_K. The Jacobi variety is isomorphic to X_K and therefore also possesses a strictly semi-stable model J. Let $\overline{J}$ be the identity component of the open part of J which is formally smooth over R. Then

$$\overline{J} = \overline{\mathbb{G}}_{m,R} := \varinjlim \mathbb{G}_{m,R_n}$$

where we denote the n-th infinitesimal neighborhood by $R_n := R/(\pi^{n+1})$ and $X_n := X \otimes_R R_n$. The multiplicative group $\mathbb{G}_{m,R_n}$ represents the functor $\underline{\mathrm{Pic}}^0_{X_n/R_n}$ of line bundles which have degree 0 on each irreducible component of X_0. This is due to the fact that every such line bundle on X_0 is trivial on each of the projective lines contained in X_0. Namely choose local sections of the line bundle on each of these projective lines X_0^σ, i.e. meromorphic functions f_σ, which neither have poles nor zeroes in the double points. Starting on one component X_0^1 we can multiply the meromorphic function f_2 on the next component X_0^2 by a constant a so that the two functions f_1 and af_2 agree at the double point $X_0^1 \cap X_0^2$. Repeating this, we go round the circle once. The ratio f_1/f_s at the last double point where the chain of projective lines closes is the gluing data that determines this line bundle. It is an element of $\mathbb{G}_{m,k}$.

The group $\overline{J}$ represents the formally smooth deformations of the trivial line bundle on X. Its rigid analytic fiber is

$$\overline{J}_{\mathrm{rig}} = \overline{\mathbb{G}}_{m,K} := \mathrm{Sp}\, K\langle \zeta, \zeta^{-1} \rangle = \{ z \in K^{\mathrm{alg}} : |z| = 1 \} / \, \mathrm{Gal}\,.$$

It can be embedded into the multiplicative group

$$\overline{J}_{\mathrm{rig}} = \overline{\mathbb{G}}_{m,K} \hookrightarrow \mathbb{G}_{m,K} =: \widehat{J}_K\,.$$

We interpret $\widehat{J}_K$ in the following way. We have $\mathrm{H}^1(X_K, \mathbb{Z}) = \mathrm{H}^1(X_0, \mathbb{Z}) = \mathbb{Z} \cdot b$. This induces the multiplicative line bundle (ζ^b) on $X_K \times_K \mathbb{G}_{m,K}$ which gives rise to a morphism $\mathbb{G}_{m,K} \longrightarrow J_K$. The group $\widehat{J}_K$ contains a lattice

$$\Lambda := \{ \zeta \in \widehat{J}_K : \ (\zeta^b) \text{ is trivial } \}\,.$$

One can show that the quotient of $\widehat{J}_K$ by Λ is J_K; cf. [BL, Theorem 1.2].

In the general case we construct the Picard variety in a similar manner. We consider the infinitesimal neighborhoods $R_n := R/(\pi^{n+1})$ and $X_n := X \otimes_R R_n$. By assumption the scheme X_n is proper and flat over R_n with geometrically reduced special fiber. The point $x \in X(R)$ induces a point $x_n \in X_n(R_n)$ for all $n \in \mathbb{N}$. Due to the classical result of M. Artin [Ar 1, Theorem 7.3] the functor $\underline{\mathrm{Pic}}_{X_n/R_n}$ is representable by an algebraic space locally of finite type over R_n. This is a group scheme over R_n since an algebraic group space over an Artinian base is a scheme; cf. [Ar 2, Theorem 3.5]. It is locally of finite type over R_n. Let P'_n be its identity component

$$P'_n := \mathrm{Pic}^0_{X_n/R_n}\,.$$

It is a group scheme of finite type over R_n; cf. [SGA 3, I, Exposé $\mathrm{VI_A}$, Proposition 2.4]. Furthermore we have the Poincaré bundle $\mathcal{P}'_n$ on $X_n \times_{R_n} P'_n$. We set

$$P' := \varinjlim P'_n\,.$$

The limit is defined via the projections $P'_{n+1} \to P'_n$; notice that the identity $X_{n+1} \otimes_{R_{n+1}} R_n = X_n$ implies $P'_{n+1} \otimes_{R_{n+1}} R_n = P'_n$. The formal scheme P' is of topological finite type over R (cf. [FRG, I]). The system of the Poincaré bundles $\mathcal{P}'_n$ is compatible and gives rise to the Poincaré bundle $\mathcal{P}' := \varprojlim \mathcal{P}'_n$ on $X \times_R P'$.

We investigate the structure of P' . Its special fiber P'_0 does not contain an additive group $\mathbb{G}_{a,\overline{k}}$ where $\overline{k}$ is an algebraic closure of k . The reason for this lies in the semi-stability of X . Namely every line bundle on $X_0 \times_k \mathbb{G}_{a,\overline{k}}$ which is trivialized along 0 is already globally trivial. In particular the torsion points of order prime to $\operatorname{char} k$ are dense in P'_0 . However P' may not be smooth, in fact it will in general not even be flat over R . By dividing out its π-torsion and its nilpotent structure we obtain an admissible formal R-scheme $\overline{P'}$. Its rigid analytic fiber is smooth but maybe not connected. The smoothness comes from the fact that the mentioned torsion points of order prime to $\operatorname{char} k$ lift to $\overline{P'}$. We now consider the identity component $\overline{P}_K$ of $\overline{P'}_{\mathrm{rig}}$ and let $\overline{P}$ be the formal Néron model of $\overline{P}_K$; cf. [BS, Theorem 1.2]. It is a smooth formal group scheme over R and can be obtained as the identity component of the group smoothening of $\overline{P'}$; cf. [BLR, Theorem 7.1/5]. We have the following diagram

$$\begin{array}{ccccc} \overline{P}_K & \hookrightarrow & \overline{P} & & \\ \downarrow & & \downarrow & & \\ \overline{P'}_{\mathrm{rig}} & \hookrightarrow & \overline{P'} & \hookrightarrow & P' \end{array}$$

The vertical arrow on the left is a finite morphism, so also the vertical arrow on the right is finite; cf. [BGR, Theorem 6.3.5/1]. By pulling back the line bundle $\mathcal{P}'$ we obtain the Poincaré bundle $\overline{\mathcal{P}}$ on $X \times_R \overline{P}$. The group $\overline{P}$ represents the formally smooth deformations of the trivial line bundle on X , because every morphism from a smooth connected admissible formal R-scheme into P' factors through $\overline{P}$ due to the universal property of the Néron model.

Since P'_0 contains no additive group and $\overline{P} \longrightarrow P'$ is finite, the unipotent part of the special fiber $\overline{P}_0$ is trivial [SGA 3, II, Exposé XVII, Proposition 4.1.1]. This implies that $\overline{P}$ is semi-abelian, i.e.

$$1 \longrightarrow \overline{T} \longrightarrow \overline{P} \longrightarrow B \longrightarrow 1$$

the formal group scheme $\overline{P}$ is an extension of a formal abelian scheme B by a formal torus $\overline{T} \cong \overline{\mathbb{G}}^r_{m,R} = \varinjlim \mathbb{G}^r_{m,R_n}$; cf. [BLR, Theorem 9.2.1] and [SGA 3, II, Exposé XVII, Théorème 7.2.1]. The torus part accounts for the multiplicative line bundles, i.e. $\mathrm{H}^1(X_0, \mathbb{Z}) \cong \mathrm{Hom}(\overline{\mathbb{G}}_{m,R}, \overline{T})$.

The formal torus $\overline{T}_{\mathrm{rig}} = \overline{\mathbb{G}}^r_{m,K}$ is embedded into the affine Torus $T_K := \mathbb{G}^r_{m,K}$. We define $\widehat{P}_K$ to be the push-out of $\overline{P}_K$ via this embedding

$$\begin{array}{ccccccccc} 1 & \longrightarrow & \overline{T}_{\mathrm{rig}} & \longrightarrow & \overline{P}_K & \longrightarrow & B_{\mathrm{rig}} & \longrightarrow & 1 \\ & & \downarrow & & \downarrow & & \| & & \\ 1 & \longrightarrow & T_K & \longrightarrow & \widehat{P}_K & \longrightarrow & B_{\mathrm{rig}} & \longrightarrow & 1 . \end{array}$$

Due to its cubical structure the Poincaré bundle $\overline{\mathcal{P}}_{\mathrm{rig}}$ on $X_K \times_K \overline{P}_K$ extends to a line bundle $\widehat{\mathcal{P}}_K$ on $X_K \times_K \widehat{P}_K$; cf. [Mo, Définition I.2.4.5] and [BL, Propositions

4.1 and 4.2]. We will now construct the Picard variety as a quotient of $\widehat{P}_K$ by a suitable lattice. Let

$$\Lambda := \{ p \in \widehat{P}_K : \ (\mathrm{id}_{X_K} \times p)^* \widehat{\mathcal{P}}_K \text{ is trivial} \}.$$

Then Λ is a lattice in $\widehat{P}_K$ and the quotient $P_K := \widehat{P}_K/\Lambda$ is a smooth rigid analytic group variety. The lattice Λ also operates canonically on the Poincaré bundle $\widehat{\mathcal{P}}_K$. We can divide out this action and obtain the Poincaré bundle $\mathcal{P}_K$ on $X_K \times_K P_K$. We claim

Theorem 3.1 *The smooth connected rigid analytic group variety P_K represents the functor $\underline{\mathrm{Pic}}^0_{X_K/K}$ on the category $\mathfrak{C}_K$.*

Proof 1.) Let first be V a formal scheme as in Proposition 2.1 and $\mathcal{L}_K$ a line bundle on $X_K \times_K V_{\mathrm{rig}}$ trivialized along $v \in V_{\mathrm{rig}}(K)$. Then $\mathcal{L}_K$ decomposes uniquely according to Proposition 2.1

$$\mathcal{L}_K \cong \mathcal{M}_K \otimes \mathcal{N}_{\mathrm{rig}}$$

into a multiplicative line bundle $\mathcal{M}_K$ and a formal line bundle $\mathcal{N}$. The line bundle $\mathcal{M}_K$ gives rise to a uniquely determined morphism

$$V_{\mathrm{rig}} \longrightarrow T_K \longrightarrow \widehat{P}_K \longrightarrow P_K .$$

The line bundle $\mathcal{N}$ gives rise to a uniquely determined morphism

$$V_{\mathrm{rig}} \longrightarrow \overline{P}_K \longrightarrow \widehat{P}_K \longrightarrow P_K .$$

By adding these two morphisms in P_K we get a uniquely determined morphism $f : V_{\mathrm{rig}} \longrightarrow P_K$ with $\mathcal{L}_K \cong (\mathrm{id}_{X_K} \times f)^* \mathcal{P}_K$.

2.) Now let V_K be an arbitrary smooth rigid analytic space and $\mathcal{L}$ a line bundle on $X_K \times_K V_K$ which is trivialized along some $v \in V_K(K)$. By the formal analog of de Jong's alteration result there exists an étale covering $\widetilde{V}_K \longrightarrow V_K$ which admits a semi-stable formal model $\widetilde{V}$. Further we can chose $\widetilde{V}$ to be of the type considered under 1.) and so we obtain a morphism $\widetilde{f} : \widetilde{V}_K \longrightarrow P_K$. We can descend this morphism to the morphism $f : V_K \longrightarrow P_K$ with $\mathcal{L}_K \cong (\mathrm{id}_{X_K} \times f)^* \mathcal{P}_K$ we were looking for. □

The rigid analytic group P_K will be the identity component of the representing space for the usual Picard functor $\underline{\mathrm{Pic}}_{X_K/K}$.

As we have remarked in the beginning (Remark 1.6), P_K need not be proper. Nevertheless it is an extension of an abeloid variety by an affine torus. Namely using the lattice Λ we can split the torus T_K of $\widehat{P}_K$ into a product of subtori

$$T_K = T'_K \times T''_K$$

with $\mathrm{rk}_{\mathbb{Z}} \Lambda = \dim T''_K = r''$. Due to the structure of $\widehat{P}_K$ we obtain an extension via push-out with respect to the projection $T_K \longrightarrow\!\!\!\!\rightarrow T''_K$

$$\begin{array}{ccccccccc} 1 & \longrightarrow & T_K & \longrightarrow & \widehat{P}_K & \longrightarrow & B_K & \longrightarrow & 1 \\ & & \downarrow\!\!\!\downarrow & & \downarrow\!\!\!\downarrow & & \| & & \\ 1 & \longrightarrow & T''_K & \longrightarrow & \widehat{Q}_K & \longrightarrow & B_K & \longrightarrow & 1 . \end{array}$$

The canonically induced map $\widehat{P}_K \twoheadrightarrow \widehat{Q}_K$ gives rise to an isomorphism of lattices

$$\begin{array}{ccc} \widehat{P}_K & \twoheadrightarrow & \widehat{Q}_K \\ \uparrow & & \uparrow \\ \Lambda & \xrightarrow{\sim} & \Lambda'' \end{array}$$

where Λ'' is now a lattice of $\widehat{Q}_K$ of full rank r''. In particular, the quotient

$$Q_K = \widehat{Q}_K / \Lambda''$$

is an abeloid variety, i.e. a smooth proper connected rigid analytic group variety. Thus we obtain the following commutative diagram

$$\begin{array}{ccccccccc} & & & & \Lambda & = & \Lambda'' & & \\ & & & & \downarrow & & \downarrow & & \\ 1 & \longrightarrow & T'_K & \longrightarrow & \widehat{P}_K & \longrightarrow & \widehat{Q}_K & \longrightarrow & 1 \\ & & \| & & \downarrow & & \downarrow & & \\ 1 & \longrightarrow & T'_K & \longrightarrow & P_K & \longrightarrow & Q_K & \longrightarrow & 1 \end{array}$$

which makes P_K an extension of an abeloid variety Q_K by an affine torus T'_K. This proves Theorem 1.5 as far as the identity component is concerned.

4 The Néron-Severi Group

We next study the Néron-Severi group of X_K which is by definition the quotient

$$\mathrm{NS}_{X_K/K} = \underline{\mathrm{Pic}}_{X_K/K}(\mathbb{K}) \,/\, \mathrm{Pic}^0_{X_K/K}(\mathbb{K})$$

where $\underline{\mathrm{Pic}}_{X_K/K}(\mathbb{K})$ denotes the group of isomorphism classes of line bundles on $X_K \widehat{\otimes}_K \mathbb{K}$ and where $\mathrm{Pic}^0_{X_K/K}(\mathbb{K})$ is the set of $\mathbb{K}$-rational points of P_K. We want to relate this group to the Néron-Severi group of the special fiber X_0; i.e., to the group

$$\mathrm{NS}_{X_0/k} = \mathrm{Pic}_{X_0/k}(\overline{k}) \,/\, \mathrm{Pic}^0_{X_0/k}(\overline{k})$$

where $\overline{k}$ is an algebraic closure of the residue field k of R. Therefore we note that every element of $\mathrm{NS}_{X_K/K}$ can in fact be represented by a line bundle $\mathcal{L}_K$ on $X_K \otimes_K \widetilde{K}$ for a finite field extension $\widetilde{K}$ of K. We try to extend $\mathcal{L}_K$ to a formal line bundle $\mathcal{L}$ on $\widetilde{X} := X \otimes_R \widetilde{R}$ where $R \subseteq \widetilde{R}$ is the associated extension of discrete valuation rings. If $\widetilde{\pi}$ is the uniformizer of $\widetilde{R}$ and $e \in \mathbb{N}$ the ramification index then

$$\pi = \widetilde{u} \cdot \widetilde{\pi}^{\,e}$$

for a unit $\widetilde{u}$ of $\widetilde{R}$. In general the formal line bundle $\mathcal{L}$ does not exist, since $\widetilde{X}$ is not regular. But we can proceed similarly to the case of $X \times_R V$ we discussed in Proposition 2.1:

(a) There is a desingularization procedure to obtain a regular formal model $\widetilde{X} \longleftarrow \widetilde{Y}$ of $\widetilde{X}_{\rm rig} = \widetilde{Y}_{\rm rig}$. The line bundle $\mathcal{L}_K$ extends to a formal line bundle $\widetilde{\mathcal{L}}$ on $\widetilde{Y}$.

(b) With respect to a suitable covering $\{U^i\}$ of X the line bundle $\widetilde{\mathcal{L}}$ descends to line bundles $\mathcal{L}^i$ on $\widetilde{U}^i := U^i \otimes_R \widetilde{R}$.

(c) In order to glue these local extensions of $\mathcal{L}_K$ we study the invertible functions on the intersections $\widetilde{W} := \widetilde{U} \cap \widetilde{U}'$.

This time $\mathcal{O}_{\widetilde{X}_{\rm rig}}(\widetilde{W}_{\rm rig})^\times$ contains the following subgroup

$$\mathcal{O}_{\widetilde{X}_{\rm rig}}(\widetilde{W}_{\rm rig})^\times \supseteq \mathcal{O}_{\widetilde{X}}(\widetilde{W})^\times \oplus \xi_1^{\Gamma(\widetilde{W}_0,\mathbb{Z})} \oplus \ldots \oplus \xi_s^{\Gamma(\widetilde{W}_0,\mathbb{Z})} .$$

The quotient by this subgroup is $\Gamma(\widetilde{W}_0,\, \mathbb{Z}/e\mathbb{Z})$, coming from the fact that $\widetilde{\pi}$ is an invertible function on $\widetilde{W}_{\rm rig}$. This means we can associate to $\mathcal{L}_K$ an element

$$\mathrm{cyc}(\mathcal{L}_K) \in \mathrm{H}^1(X_0,\, \mathbb{Z}/e\mathbb{Z}) .$$

We consider the exact sequence

$$0 \longrightarrow \mathbb{Z} \longrightarrow \mathbb{Z} \longrightarrow \mathbb{Z}/e\mathbb{Z} \longrightarrow 0$$

of constant sheaves on X_0 . The image of $\mathrm{cyc}(\mathcal{L}_K)$ under the connecting homomorphism $\delta : \mathrm{H}^1(X_0,\, \mathbb{Z}/e\mathbb{Z}) \longrightarrow \mathrm{H}^2(X_0,\mathbb{Z})$ is zero if and only if $\mathrm{cyc}(\mathcal{L}_K)$ is the image of some $n \in \mathrm{H}^1(X_0,\mathbb{Z})$. This is the case if and only if there exists a multiplicative line bundle $\mathcal{M}_K := (\widetilde{\pi}^n)$ on $\widetilde{X}_{\rm rig}$ such that $\mathcal{L}_K \otimes \mathcal{M}_K^\vee$ extends to a formal line bundle $\mathcal{L}$ on $\widetilde{X}$. Since $\mathcal{M}_K$ corresponds to a $\widetilde{K}$-rational point of $\widehat{P}_K$ it plays no role in the investigation of the Néron-Severi group. So we see that the obstruction for extending $\mathcal{L}_K$ globally to a line bundle $\mathcal{L}$ on $\widetilde{X}$ is given by an element of

$$\mathrm{im}\Big(\delta : \mathrm{H}^1(X_0,\, \mathbb{Z}/e\mathbb{Z}) \longrightarrow \mathrm{H}^2(X_0,\mathbb{Z})\Big),$$

i.e. by a torsion element of $\mathrm{H}^2(X_0,\mathbb{Z})$. Since the later cohomology group is finitely generated, it contains only finitely many torsion elements.

The line bundles for which this obstruction vanishes form a subgroup $\mathrm{NS}^1_{X_K/K}$ of $\mathrm{NS}_{X_K/K}$. By reducing mod π we get a map

$$\mathrm{NS}^1_{X_K/K} \longrightarrow \mathrm{NS}_{X_0/k} \;/\; M$$

where M is the image of the set of formal line bundles on $\widetilde{X}$ which induce multiplicative line bundles on the generic fiber. Elements in the kernel of this map can be represented by formal line bundles $\mathcal{L}$ on $\widetilde{X}$ such that the reduction $\mathcal{L}_0 := \mathcal{L} \otimes_{\widetilde{R}} \widetilde{k}$ belongs to $\mathrm{Pic}^0_{X_0/k}(\widetilde{k})$. There $\widetilde{k}$ is the residue field of $\widetilde{R}$. Due to our construction, the class of $\mathcal{L}$ is therefore an $\widetilde{R}$-rational point of $\overline{P'}$. Thus we obtain an exact sequence

$$\overline{P'}_{\rm rig}(\mathbb{K}) \,/\, \overline{P}_K(\mathbb{K}) \longrightarrow \mathrm{NS}^1_{X_K/K} \longrightarrow \mathrm{NS}_{X_0/k} \;/\; M$$

Since $\overline{P}_K$ is a subgroup of finite index in $\overline{P'}_{\rm rig}$ we conclude that $\mathrm{NS}^1_{X_K/K}$ is an extension of a subquotient of the Néron-Severi group of the special fiber by a finite group. $\mathrm{NS}_{X_0/k}$ is finitely generated; cf. [SGA 6, Exposé XIII, Thm. 5.1]. Thus the Néron-Severi group $\mathrm{NS}_{X_K/K}$ of X_K is finitely generated.

The rigid analytic group variety

$$\mathrm{Pic}_{X_K/K} \;:=\; \coprod_{[\mathcal{L}_K]\,\in\,\mathrm{NS}_{X_K/K}} \mathcal{L}_K \otimes \mathrm{Pic}^0_{X_K/K}$$

represents the Picard functor $\underline{\mathrm{Pic}}_{X_K/K}$ on the category of smooth rigid analytic spaces. This finishes the proof of Theorem 1.5. □

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