

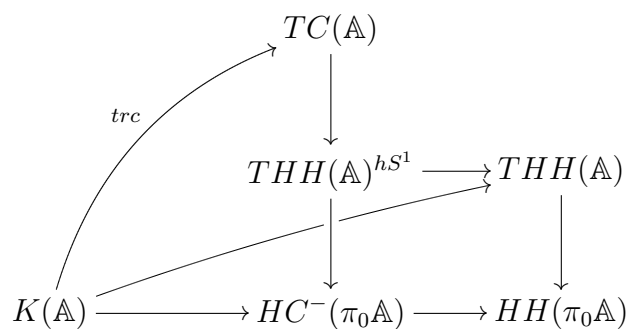
Workshop on

Trace Methods in

Algebraic K -Theory

MÜNSTER, GERMANY

September 29 – October 3, 2003



Abstracts

CHRISTIAN AUSONI

Topological cyclic homology of topological K -theory

In these talks, I would like to outline a computation of the topological cyclic homology of the connective Adams summand (joint work with John Rognes) and of connective complex K -theory. I will consider in parallel the cases of \mathbb{F}_p (Hesselholt-Madsen) and \mathbb{Z} (Bökstedt-Madsen), highlighting the similarities between these various computations.

ARTHUR BARTELS

On the Farrell-Jones conjecture for higher algebraic K -theory

(joint work with Holger Reich)

We prove the Farrell-Jones conjecture about the algebraic K -theory of a group ring $R\Gamma$ in the case where the group Γ is the fundamental group of a closed Riemannian manifold with strictly negative sectional curvature. The coefficient ring R is an arbitrary associative ring with unit and the result applies to all dimensions.

SPENCER BLOCH

to be announced

VIOREL COSTEANU

The 2-typical de Rham-Witt complex of the integers

WOJTEK DORABIALA

Trace and transfer maps in the algebraic K -theory of spaces

We investigate the relationship between the transfer map in the algebraic K -theory of spaces and the Becker-Gottlieb transfer. The result is that for a general class of topological fiber bundles, A -theory transfer followed by the trace map is homotopy equivalent to the Becker-Gottlieb transfer.

HÉLÈNE ESNAULT

Additive Chow groups and some Chow groups with modulus

(joint work with Spencer Bloch)

Spencer Bloch defined higher Chow groups on schemes. They generalize the more classical notion of Chow groups for schemes, as well as Milnor K -theory for fields. In an attempt to put a modulus 2 condition on those higher Chow groups, we defined an additive version of the higher Chow groups. For fields, they compute absolute differential forms. The hope is that those groups with higher modulus will be the E_2 term of a spectral sequence converging to additive K -theory as defined in the work of Hesselholt (and coauthors), just as Bloch's higher Chow groups are the E_2 term of a spectral sequence converging to K -theory. The aim of the lecture will be to motivate this construction by showing the link to generalized jacobians, and more generally to Chow groups with modulus. We will also show how a sort of secondary class in Hodge theory yields the correct differential forms.

THOMAS GEISSER

The cyclotomic trace map and special values of zeta functions

I will give a short survey of joint results with Lars Hesselholt on topological cyclic homology of varieties over henselian discrete valuation rings. As an application, I explain how to relate the cokernel of the cyclotomic trace map from algebraic K -theory to topological cyclic homology to special values of zeta functions in some situations.

LARS HESSELHOLT

Topological Hochschild homology and the de Rham-Witt complex

First talk: In this talk, I will introduce the de Rham-Witt complex and the Witt complex TR associated with topological Hochschild homology. I will also prove an explicit formula that expresses these constructions for a Laurent polynomial algebra in terms of the corresponding constructions for the ring of coefficients.

Second talk: I will outline a variety of calculations of the Witt complex TR . In all cases, the relationship of the de Rham-Witt complex and TR is similar to that of Milnor K -theory and algebraic K -theory. By analogy, one is led to conjecture that exist a "motivic" theory related to TR . In the end, I will explain how the calculations can be understood from this conjectural picture.

Third talk: In the final talk, I will present a candidate for a "motivic" theory for TR . It is related to the additive higher Chow groups of Bloch and Esnault. I will also explain how this theory is related to p -adic vanishing cycles.

RANDY MCCARTHY

**The Witt spectrum of a bimodule and
its relation to continuous algebraic K -theory**

We will define a topological spectrum for an R -bimodule M , $W(R, M)$, which has structural relations like the Witt ring of a commutative ring. In particular, $W(R, R)$ is simply $TR(R)$ from topological cyclic homology and hence the fundamental group of $W(R, R)$ is Lars Hesselholt's Witt module for a not necessarily commutative ring R . We will sketch a fairly simple proof using Goodwillie's calculus of functors that for M a projective R -bimodule $\operatorname{holim} K(T_R(M)/I^n) \simeq W(R, M)$ where $T_R(M)$ is the tensor algebra of M and I is the augmentation ideal of $T_R(M)$ with respect to R .

HOLGER REICH AND MARCO VARISCO

Assembly maps and isomorphism conjectures

The algebraic K -theory of group rings (or group \mathbb{S} -algebras) plays an important role in the study of high-dimensional manifolds. The Farrell-Jones isomorphism conjecture predicts that these K -groups are built up from group-homological information and the algebraic K -theory of group algebras of small subgroups. This idea can be precisely formulated using the general machinery of assembly maps, which also applies to other contexts and in particular to topological Hochschild homology and topological cyclic homology of group algebras.

After a general introduction to assembly maps and a brief survey on known results about the Farrell-Jones isomorphism conjecture, our main goal is to show how the comparison of the K -theory and TC assembly maps via the cyclotomic trace can lead to rational injectivity results for the Farrell-Jones assembly map. This method goes back to the work of Bökstedt, Hsiang, and Madsen on the algebraic K -theory Novikov conjecture. In the lectures we will describe a simplified version of their argument and explain how we applied this (in joint work with Wolfgang Lück and John Rognes) to more general assembly maps. We are naturally led to consider the effect in algebraic K -theory of the completion map for rings of integers in cyclotomic fields, and therefore to the Leopoldt conjecture and its higher analogs. Another important ingredient are general splitting and isomorphism results about THH and TC assembly maps, which are also of independent interest.

CHRISTIAN SCHLICHTKRULL

Topological Hochschild homology of Thom spectra

(joint work with Ralph Cohen)

In homotopy theory, the suspension spectrum of a loop space is often viewed as a topological analogue of the integral group ring. Similarly, a Thom spectrum may

be viewed as a twisted group ring. Examples of such are the classical Thom spectra MO , MU , etc., and, by work of Mahowald, the Eilenberg-MacLane spectra $H\mathbb{Z}$ and $H\mathbb{Z}/p$.

In this talk we present a general formula for the topological Hochschild homology of a Thom spectrum. The answer is in terms of a certain Thom spectrum over the free loop space, and provides a unifying approach to the basic calculations of topological Hochschild homology.

BRUCE WILLIAMS

Parameterized fixed point theory

Suppose M is a compact smooth manifold. Then the Euler characteristic has five equivalent descriptions.

1. Algebraic: $\sum (-1)^i \dim H_i(M; \mathbb{R})$.
2. Obstruction Theory: Poincaré dual of the Euler class of the tangent bundle.
3. Stable Homotopy: The degree of the stable map

$$S \rightarrow M_+ \wedge DM_+ \xrightarrow{\Delta \wedge 1} M_+ \wedge M_+ \wedge DM_+ \rightarrow M_+ \wedge S \rightarrow S.$$

4. Bordism: Transversal self intersection of the zero section of the tangent bundle.
5. Analytic: Index of the Euler operator.

Given a family of manifolds, i.e. a smooth bundle $p: E \rightarrow B$ with compact fibers, $p^{-1}(b) = F_b$, one again gets five related parameterized Euler characteristics.

1. Algebraic: A continuous rule $\{b \in B\} \mapsto \chi(F_b) \in A(F_b)$, where $A(F_b)$ is Waldhausen's algebraic K-theory of the space F_b .
2. Obstruction Theory: Fiberwise Poincaré dual of the Becker-Euler class of the tangent bundle along the fiber.
3. Stable Homotopy: The Becker-Gottlieb-Dold homotopy transfer for p .
4. Bordism: Transversal self intersection of the zero section of the tangent bundle along the fiber.
5. Analytic: Index of the fiberwise Euler operator.

If the smooth fiber bundle is equipped with an endomorphism φ then it is reasonable to expect a Lefschetz type analogue of the above. This talk is a report on progress made in realizing this expectation. A key tool is a homotopy theoretic approach to intersection theory.

THOMAS ZINK

Display structure in the cohomology of the de Rham Witt-complex

Schedule

Monday, September 29

- 10:00–11:00 SPENCER BLOCH
to be announced
- 11:45–12:45 LARS HESSELHOLT
Topological Hochschild homology and the de Rham-Witt complex
(I of III)
- 14:30–15:30 CHRISTIAN SCHLICHTKRULL
Topological Hochschild homology of Thom spectra
- 16:30–17:30 HOLGER REICH
Assembly maps and isomorphism conjectures (I of III)

Tuesday, September 30

- 10:00–11:00 LARS HESSELHOLT
Topological Hochschild homology and the de Rham-Witt complex
(II of III)
- 11:45–12:45 WOJTEK DORABIALA
Trace and transfer maps in the algebraic K -theory of spaces
- 14:30–15:30 MARCO VARISCO
Assembly maps and isomorphism conjectures (II of III)
- 16:30–17:30 THOMAS GEISSER
The cyclotomic trace map and special values of zeta functions

Wednesday, October 1

- 10:00–11:00 HÉLÈNE ESNAULT
Additive Chow groups and some Chow groups with modulus
- 11:45–12:45 HOLGER REICH
Assembly maps and isomorphism conjectures (III of III)

Thursday, October 2

- 10:00–11:00 LARS HESSELHOLT
Topological Hochschild homology and the de Rham-Witt complex
(III of III)
- 11:45–12:45 ARTHUR BARTELS
On the Farrell-Jones conjecture for higher algebraic K -theory
- 14:30–15:30 CHRISTIAN AUSONI
Topological cyclic homology of topological K -theory (I of II)
- 16:30–17:30 VIOREL COSTEANU
The 2-typical de Rham-Witt complex of the integers

Friday, October 3

- 10:00–11:00 RANDY MCCARTHY
The Witt spectrum of a bimodule and its relation to continuous algebraic K -theory
- 11:45–12:45 CHRISTIAN AUSONI
Topological cyclic homology of topological K -theory (II of II)
- 14:30–15:30 THOMAS ZINK
Display structure in the cohomology of the de Rham Witt-complex
- 16:30–17:30 BRUCE WILLIAMS
Parameterized fixed point theory

ALL TALKS TAKE PLACE IN LECTURE HALL M5

Schedule

	Monday Sep 29	Tuesday Sep 30	Wednesday Oct 1	Thursday Oct 2	Friday Oct 3
9:00–10:00	<i>registration</i>				
10:00–11:00	Bloch	Hesselholt II	Esnault	Hesselholt III	McCarthy
11:00–11:45	<i>tea & coffee</i>	<i>tea & coffee</i>	<i>tea & coffee</i>	<i>tea & coffee</i>	<i>tea & coffee</i>
11:45–12:45	Hesselholt I	Dorabiala	Reich (& Varisco) III	Bartels	Ausoni II
12:45–14:30	<i>lunch</i>	<i>lunch</i>	<i>lunch</i>	<i>lunch</i>	<i>lunch</i>
14:30–15:30	Schlichtkrull	(Reich &) Varisco II	<i>excursion</i>	Ausoni I	Zink
15:30–16:30	<i>tea & coffee</i>	<i>tea & coffee</i>		<i>tea & coffee</i>	<i>tea & coffee</i>
16:30–17:30	Reich (& Varisco) I	Geisser		Costeanu	Williams